Circular motion and energy slide deck

Physics 20

Unit C: Circular motion and energy

Describing circular motion

A teacup seat holds a steady speed

- A midway teacup seat travels a horizontal circle of radius 4.50 m and finishes one full turn every 3.60 s.
- The speed stays steady while the velocity arrow swings through every compass direction around the pole.

Period and frequency

- Uniform circular motion is two dimensional motion with constant speed and a changing direction.
- Radius r, period T, and frequency f are the measurements. Frequency f equals 1 divided by T.
- Speed v equals 2 pi r / T, which is also 2 pi r f.
- Centripetal acceleration a equals v squared over r and points toward the centre.

Key terms

uniform circular motion

Uniform circular motion is two dimensional motion with constant speed and a changing direction.

centripetal acceleration

Centripetal acceleration has magnitude v squared over r and points toward the centre of the circle. ($a = v^2/r$)

period

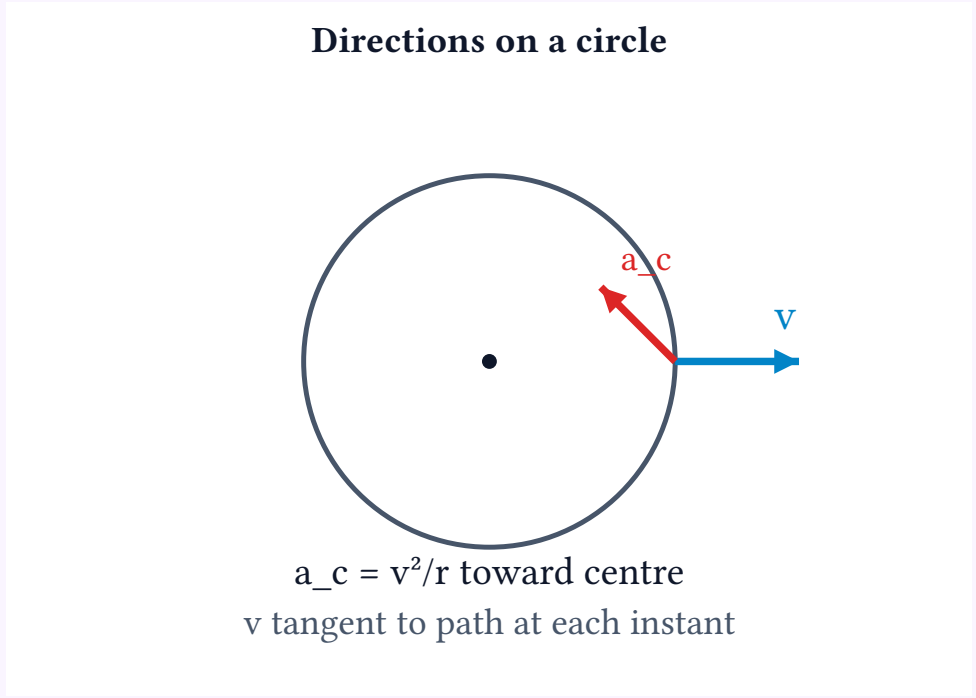
Period is the time for one full cycle, measured in seconds per cycle. (T)

frequency

Frequency is the number of cycles in one second and equals the reciprocal of the period. ($f = 1/T$)

Velocity and acceleration directions

- At each instant the velocity points along the tangent to the circle.
- Centripetal acceleration points toward the centre and has magnitude v squared over r.
- A larger radius at the same period produces a larger speed.

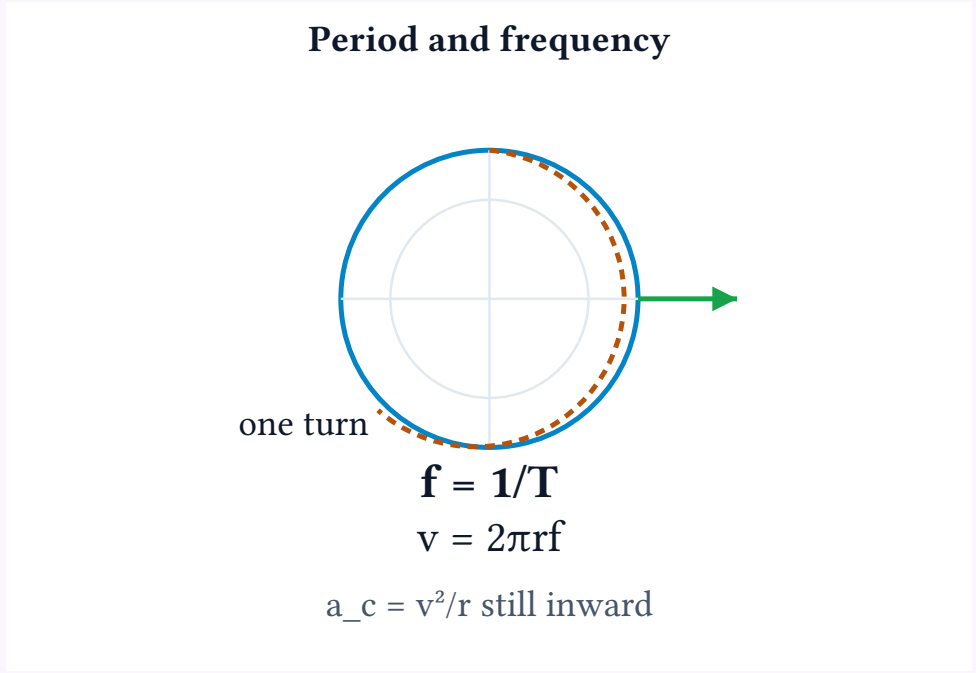


A skater’s lap on the outdoor rink

- The skater’s circle has radius 8.00 m and period 5.00 s.
- Speed = 2 × 3.14 × 8.00 m / 5.00 s = 10.1 m/s.
- With the unrounded speed, $v^2 / r = 12.6\ m/s^2$ toward the centre.

Frequency is one divided by the period

- Frequency f equals 1 divided by the period T.
- The speed can be written as 2 pi r f when the data arrive as turns each second.
- Centripetal acceleration still uses v squared over r and points toward the centre.



A go kart lap

- A go kart follows a circle of radius 25.0 m with a period of 14.0 s.
- Find the speed, the frequency, and the centripetal acceleration.
- State the direction of the acceleration.

A runner’s circular track

- A runner follows a circular track of radius 30.0 m and completes one lap in 18.0 s.
- Find the speed, the frequency, and the centripetal acceleration.
- Include the unit and the direction the acceleration requires.

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Forces in circles

A car on a curve

- A 1200 kg car holds 16.0 m/s through a horizontal curve of radius 45.0 m.
- Friction toward the centre supplies the net force that keeps changing the car’s direction.

The inward net force has a name

- In m v squared over r, m is the mass, v is the speed, and r is the radius.
- Centripetal force is the net force toward the centre, and its size is m v squared over r.
- Banking tilts the normal force so a component points toward the centre of a curve.
- A centrifuge, a carnival circle, and a potter’s wheel each need that inward net force.

Key terms

centripetal force

Centripetal force is the net force toward the centre of the circle, with magnitude m v squared over r. ($F = m\,v^2/r$)

banking

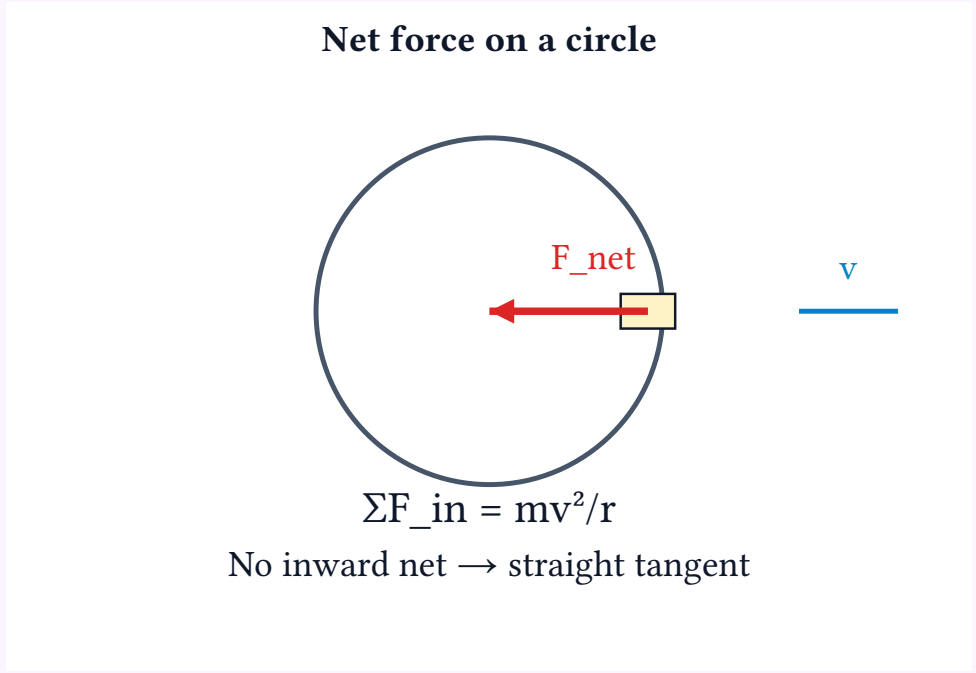
Banking tilts a roadway so the normal force has a horizontal component toward the centre of the curve.

centrifuge

A centrifuge spins samples in a circle so a large inward net force separates materials by how they respond to that force.

Newton’s laws on the circle

- The net force toward the centre equals m v squared over r.
- A zero inward net force leaves the velocity on a straight tangent.
- The inward force keeps changing the direction while it acts.

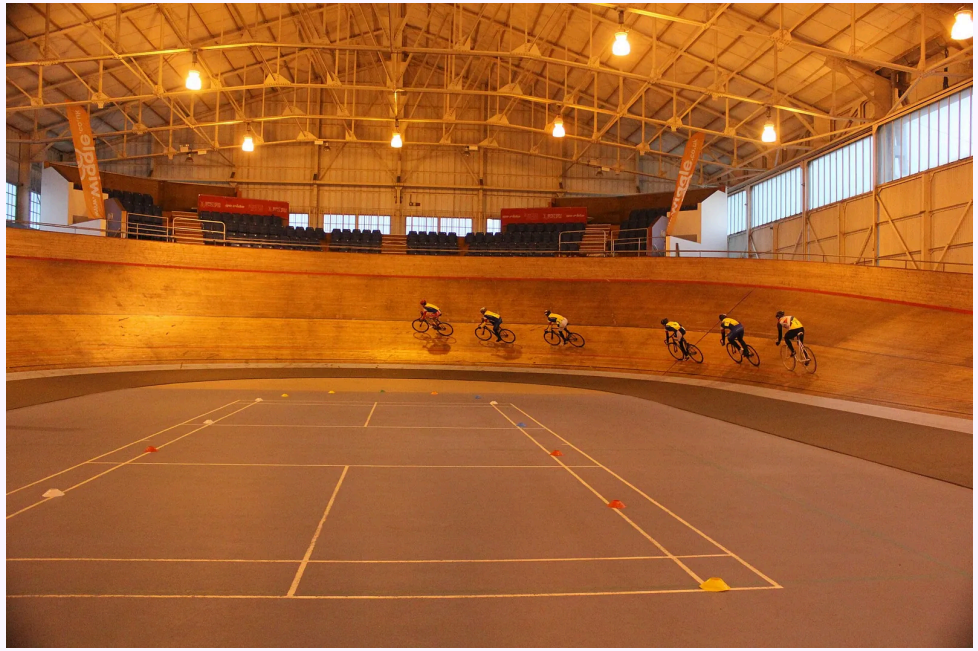


Friction on the level curve

- The car’s mass is 1200 kg, its speed is 16.0 m/s, and the radius is 45.0 m.
- $F = (1200\ \text{kg})(16.0\ \text{m/s})^2 / 45.0\ \text{m}$.
- That force is 6.83×10^3 N toward the centre.

Circular motion technologies

- A stopper lab relates orbital speed, radius, acceleration, and centripetal force.
- Banking uses an acute angle from the horizontal so the normal force can point partly inward.
- A centrifuge, a vertical carnival loop, and a potter’s wheel use the same inward net force.



Top of a vertical circle

- A 0.500 kg bucket on a 0.800 m rope moves at 4.00 m/s at the top of a vertical circle.
- At the top, tension and weight both point toward the centre.
- Their sum equals m v squared over r.

A clinical centrifuge

- A centrifuge holds a 0.00800 kg sample at radius 0.120 m and frequency 40.0 Hz.
- Find the speed and the centripetal force on the sample.
- Name the real object that supplies the inward force.

Image credits

Paul W, Calshot velodrome, CC BY-SA 4.0

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Orbits and Kepler's laws

A moonlet circles a giant

- A small satellite orbits a planet at radius 4.20 times 10 to the seventh metres with speed 3.50 times 10 to the third m/s.
- The gravitational pull toward the planet is the centripetal force, so the planet’s mass can be read from the orbit.



Orbits and Kepler’s laws

- M is the central mass, v is the orbital speed, r is the orbit radius, G is the gravity constant, and T is the period.
- For a circular orbit, the central mass equals speed squared times radius, divided by G.
- The same mass is M = 4 pi squared times r cubed, divided by G T squared.
- Kepler’s third law says the period squared grows with the radius cubed. That pattern pointed toward inverse square gravity.
- A satellite that orbits once each day stays over one longitude.

Key terms

satellite

A satellite is an object in orbit around a larger central body, held by the gravitational force toward that body.

geosynchronous orbit

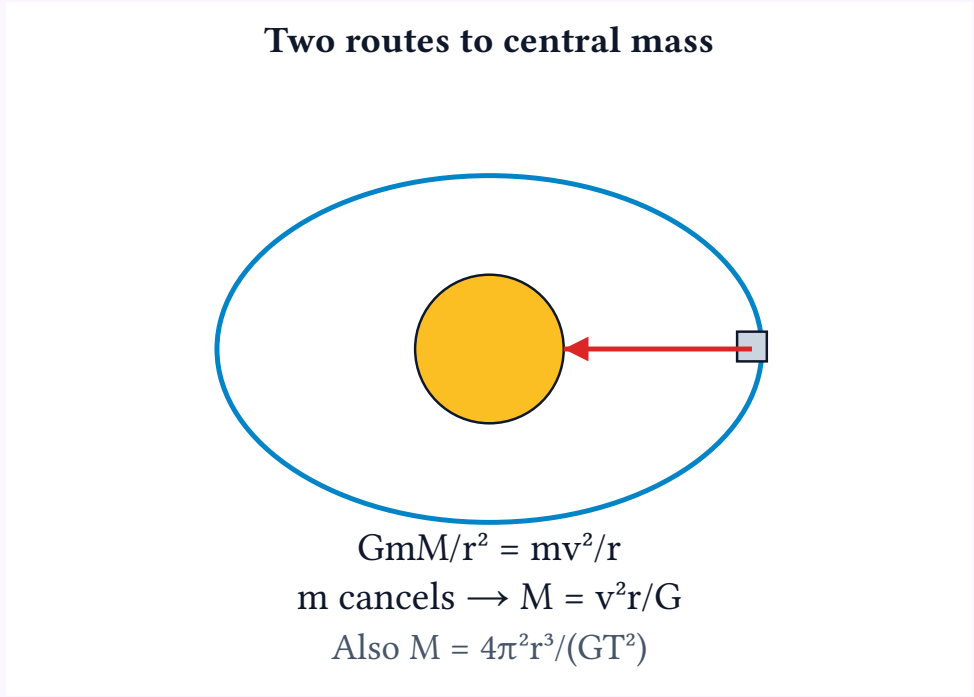
A geosynchronous orbit has a period that matches Earth's rotation period, so the satellite stays over the same longitude and can serve communication. (T = 8.64e4 s)

Kepler's laws

Kepler's laws describe orbits, and the third law says the square of the period is proportional to the cube of the average radius. (T^2 proportional to r^3)

Two routes to the central mass

- Set G M m over r squared equal to m v squared over r.
- The satellite mass m cancels, leaving $M = v^2 r / G$.
- The period form $M = 4 \pi^2 r^3 / (G T^2)$ matches that result.

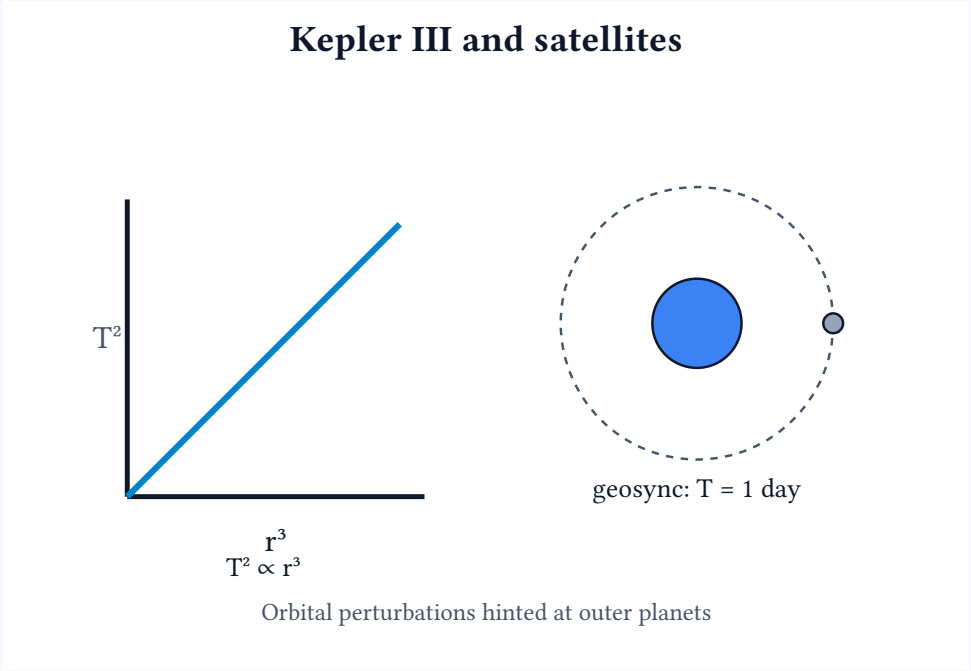


Central mass from a moonlet orbit

- Use v = 3.50 times 10 to the third m/s and r = 4.20 times 10 to the seventh metres.
- M = $(3.50\times10^{3})^{2} \times (4.20\times10^{7}) / 6.67\text{e-}11$.
- The central mass is 7.71×10^{24} kg, and the period form agrees.

Kepler’s third law and satellites

- Kepler’s third law says T squared is proportional to r cubed.
- That proportion led toward Newton’s inverse square law, and orbital perturbations later pointed to outer planets.
- A geosynchronous satellite orbits once each day, so a dish on the ground can keep pointing at it.



An outer moon’s period

- An inner moon has radius 2.00 times 10 to the eighth metres and period 2.40 times 10 to the fifth seconds.
- An outer moon of the same planet has four times the radius.
- Use Kepler’s third law to find the outer period.

Radius of a geosynchronous satellite

- Earth’s mass is 5.97 times 10 to the 24th kilograms and the rotation period used here is 8.64 times 10 to the fourth seconds.
- Find the geosynchronous radius from M = 4 pi² r³ / (G T²).
- State the communication job of that orbit.

Image credits

NASA/JPL/University of Arizona, PIA02879 - A New Year for Jupiter and Io, public domain

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Mechanical energy

A skateboarder drops in from 3.20 m

- A 48.0 kg skateboarder starts from rest at a height of 3.20 m on a quarter pipe.
- At the bottom the height is zero on the chosen scale, and the speed is 7.92 m/s.



Conservation of mechanical energy

- Em = Ek + Ep, with Em the mechanical energy, Ek the kinetic energy, and Ep the gravitational potential energy.
- Ek = (1/2) m v^2, where m is the mass and v is the speed.
- Ep = m g h, where h is the height and g is 9.81 m/s^2 downward.
- When the object keeps that energy to itself, Ek plus Ep stays the same number.

Key terms

mechanical energy

Mechanical energy is the sum of kinetic energy and gravitational potential energy for the object in the model. (Em = Ek + Ep)

kinetic energy

Kinetic energy is the energy of motion, equal to one half the mass times the speed squared. (Ek = (1/2) m v^2)

gravitational potential energy

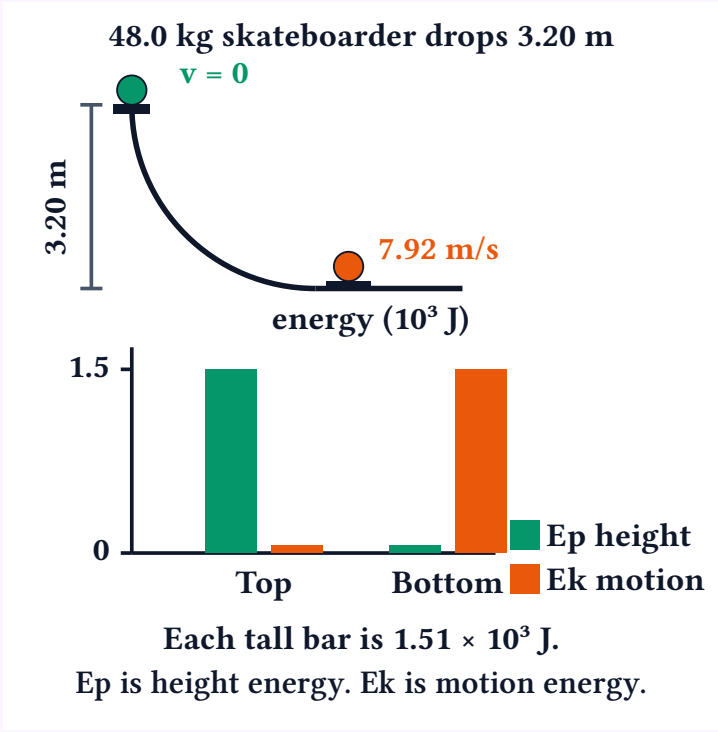
Gravitational potential energy near Earth is mass times g times height, with height measured from a chosen zero. (Ep = m g h)

isolated system

An isolated system keeps its mechanical energy constant because that energy stays inside the chosen boundary.

Energy bar charts

- At the top of a hill, gravitational potential energy holds the mechanical energy.
- At the bottom, the energy is kinetic, and the total matches the potential energy from the top.
- The bar chart keeps Ep and Ek side by side so the trade is visible.



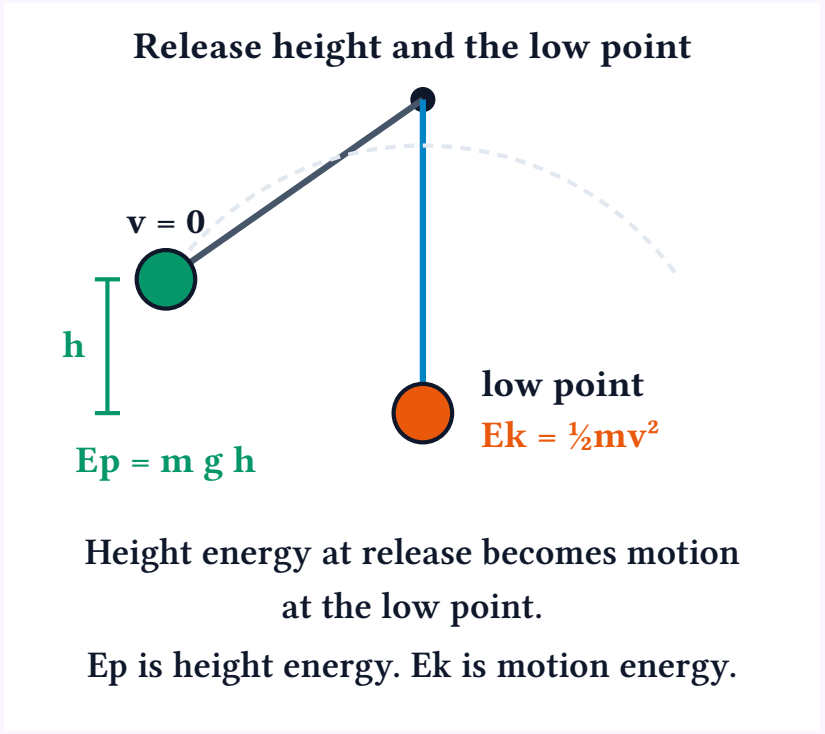
Each tall bar is 1.51 × 10³ J.
Ep is height energy. Ek is motion energy.

A ski hill checked two ways

- A 62.0 kg skier starts from rest at a height of 8.50 m. The slope length is 25.0 m.
- From the height, speed = square root of 2 × 9.81 × 8.50 = 12.9 m/s.
- The 25.0 m slope gives the same 12.9 m/s.

A pendulum lab and Joule’s paddles

- A pendulum or a rolling cart can test whether the energy total stays the same when friction is tiny.
- Release height sets Ep, and the low point converts that store into Ek.
- Joule’s falling weights and paddles linked mechanical work with thermal energy and named the joule.



Pendulum speed at the low point

- A 0.200 kg bob is released from rest 0.150 m above the low point.
- Ep at release equals m g h.
- At the low point the height energy has become kinetic energy, one half m v squared.

A toboggan hill

- A 35.0 kg toboggan starts from rest at a height of 4.80 m.
- Find Ep at the top and the speed at the bottom.
- State the quantity that stays the same from the top of the hill to the bottom.

Image credits

Tiia Monto, Skateboard ramp, CC BY-SA 3.0

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Work and power

A sled rope at 25 degrees

- A student pulls a sled with 48.0 N at 25 degrees above the horizontal through a displacement of 6.00 m.
- That pull transfers 261 J, and the same job done in 8.00 s has a power of 32.6 W.

Work and power

- W is the work, F is the force, d is the displacement, and theta is the angle between them.
- Work equals F d cos theta. P is the power, equal to W divided by delta t, and delta means final minus initial.
- W_{net} means the net work. It equals delta Ek, the change in kinetic energy.

Key terms

work

Work is the energy transferred by a force along a displacement, equal to F d cos theta. (W = F d cos theta)

power

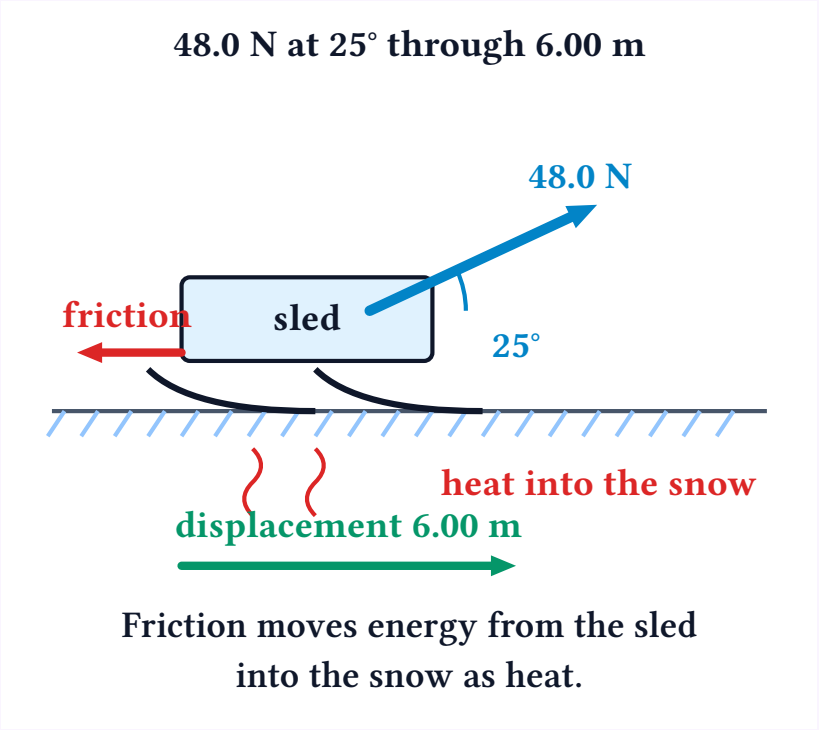
Power is the rate of doing work, equal to work divided by the elapsed time. (P = W / delta t)

work energy theorem

The work energy theorem says the net work on an object equals its change in kinetic energy. (W_{net} = delta Ek)

An open system warms the snow

- A free body diagram organizes the forces before work is calculated.
- An open system exchanges energy with the surroundings, so mechanical energy inside can change.
- Friction transfers energy into the snow and the runners as thermal energy.



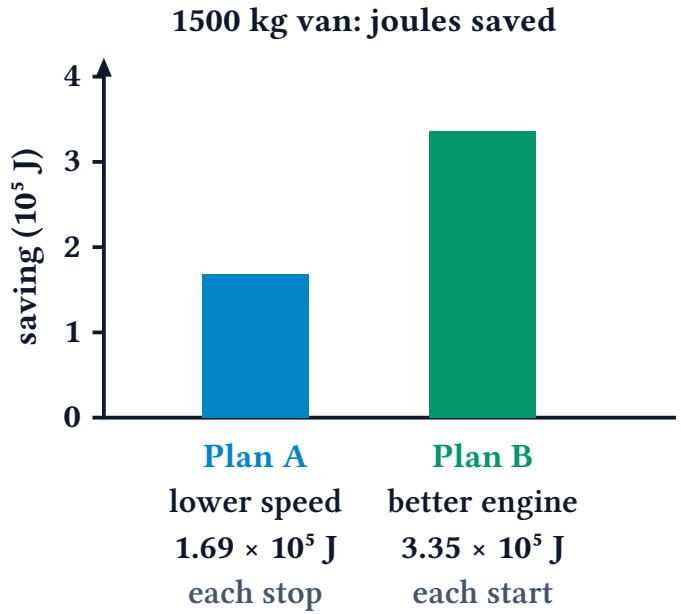
Friction moves energy from the sled into the snow as heat.

Joules and watts for the sled pull

- The pull is 48.0 N through 6.00 m at 25 degrees from the displacement.
- W = (48.0 N)(6.00 m)(cos 25 degrees) = 261 J.
- Power = 261 J / 8.00 s = 32.6 W.

Net work and kinetic energy

- W_{net}, the net work, equals delta Ek, the change in kinetic energy.
- A lower speed limit and a more efficient engine save different joules on a 1500 kg van.
- A table of the two savings, a bar for each plan, and one sentence say which plan saves more joules.



Net work equals the kinetic energy change.

Comparing energy savings

- Plan A saves 1.69 times 10 to the fifth joules of kinetic energy at each stop.
- Plan B saves 3.35 times 10 to the fifth joules of fuel energy at each acceleration to 25.0 m/s.
- State which route data would decide the better plan.

A car’s kinetic energy change

- A 1000 kg car speeds up from 10.0 m/s to 20.0 m/s over 50.0 m.
- Find delta Ek and the net force along the displacement.
- Name the theorem that links them.

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Energy transfer technologies

A meteor meets the ground

- A meteor of mass 2.40 times 10 to the fifth kilograms hits at 1.10 times 10 to the fourth m/s.
- Its kinetic energy is 1.45 times 10 to the thirteenth joules, a sudden transfer into the ground.



Efficiency and energy transfer

- A space probe swings past a moving planet and trades speed with that planet.
- Efficiency equals useful output divided by total input.
- A sustainable transfer does the job and looks after the land, the fuel, and the waste.

Key terms

gravity assist

A gravity assist is a close pass by a moving planet that exchanges momentum and energy with a spacecraft.

efficiency

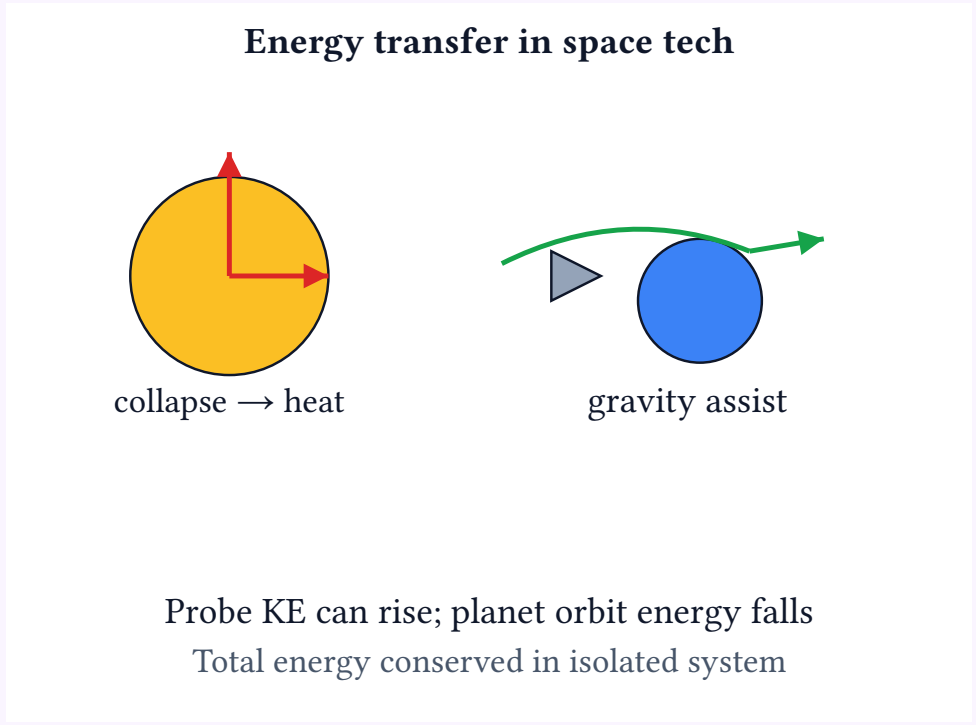
Efficiency is the useful energy output divided by the total energy input. (efficiency = useful output / total input)

sustainable energy transfer

Sustainable energy transfer meets an energy need while caring for the environment that receives the waste and supplies the source.

A collapsing shell and a planetary swing

- A collapsing star turns gravitational potential energy into thermal energy and radiation.
- A space probe swings past a moving planet and trades speed with that planet.
- The probe's kinetic energy can rise while the planet's orbital energy falls by the same amount.

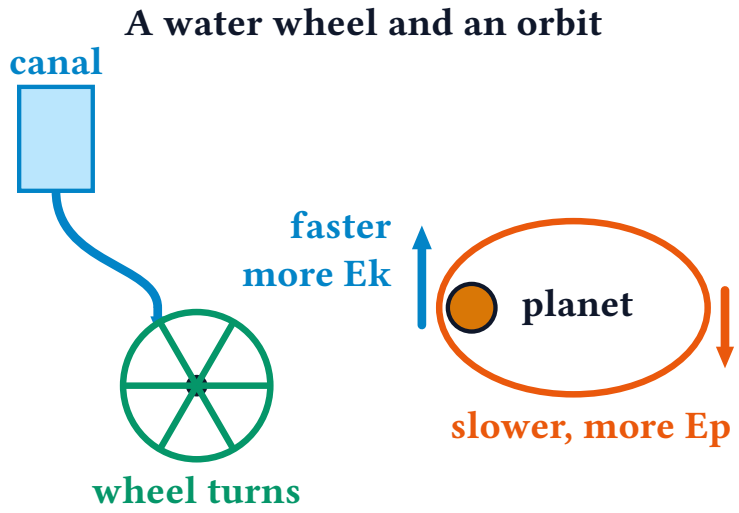


Kinetic energy of the meteor

- Mass 2.40 times 10 to the fifth kilograms, speed 1.10 times 10 to the fourth m/s.
- Ek = one half × $(2.40\times10^{5}\ \text{kg}) \times (1.10\times10^{4}\ \text{m/s})^{2}$.
- That kinetic energy is 1.45×10^{13} J into the ground.

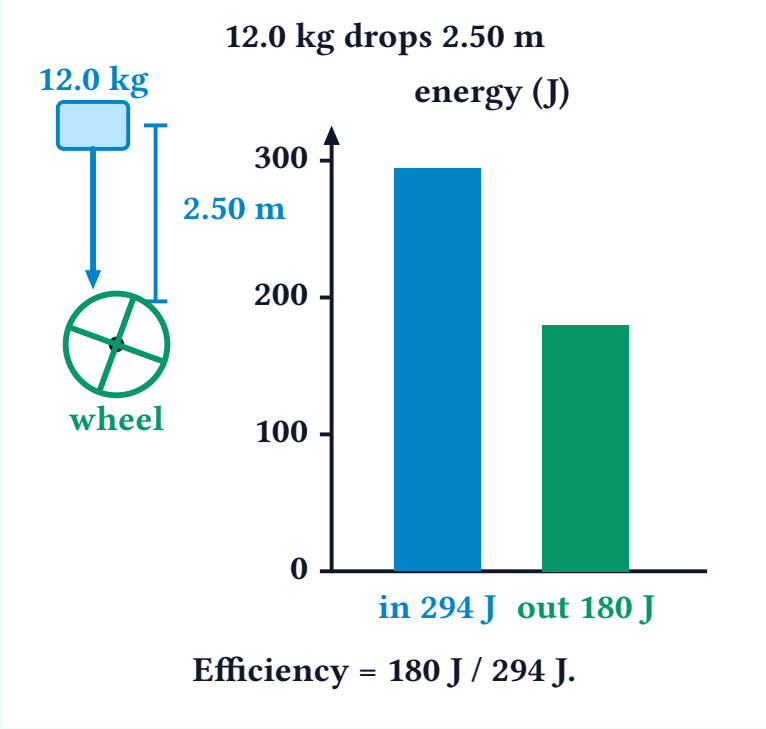
Historical energy technologies

- Closer to the planet the satellite moves faster. Farther out it moves slower, trading kinetic energy and gravitational potential energy.
- Inca terrace canals move water by gravity, and a water wheel turns that motion into rotation.
- Efficiency is useful output over total input, and an Alberta furnace or hydro plant can raise it.



Efficiency of the terrace wheel

- Canal water of mass 12.0 kg drops 2.50 m and can supply 294 J.
- The wheel delivers 180 J of rotation.
- Efficiency is useful output divided by total input.



The furnace ratio

- A home needs 64.0 MJ of useful heat.
- Compare fuel input at efficiency 0.680 with fuel input at efficiency 0.960.
- Say how the higher ratio cares for the surroundings.

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D. Roddy, U.S. Geological Survey, Barringer Meteor Crater, Arizona, public domain