Physics 20

Unit C: Circular motion and energy

Orbits and Kepler's laws

A moonlet circles a giant

- A small satellite orbits a planet at radius 4.20 times 10 to the seventh metres with speed 3.50 times 10 to the third m/s.
- The gravitational pull toward the planet is the centripetal force, so the planet’s mass can be read from the orbit.



Orbits and Kepler’s laws

- M is the central mass, v is the orbital speed, r is the orbit radius, G is the gravity constant, and T is the period.
- For a circular orbit, the central mass equals speed squared times radius, divided by G.
- The same mass is M = 4 pi squared times r cubed, divided by G T squared.
- Kepler’s third law says the period squared grows with the radius cubed. That pattern pointed toward inverse square gravity.
- A satellite that orbits once each day stays over one longitude.

Key terms

satellite

A satellite is an object in orbit around a larger central body, held by the gravitational force toward that body.

geosynchronous orbit

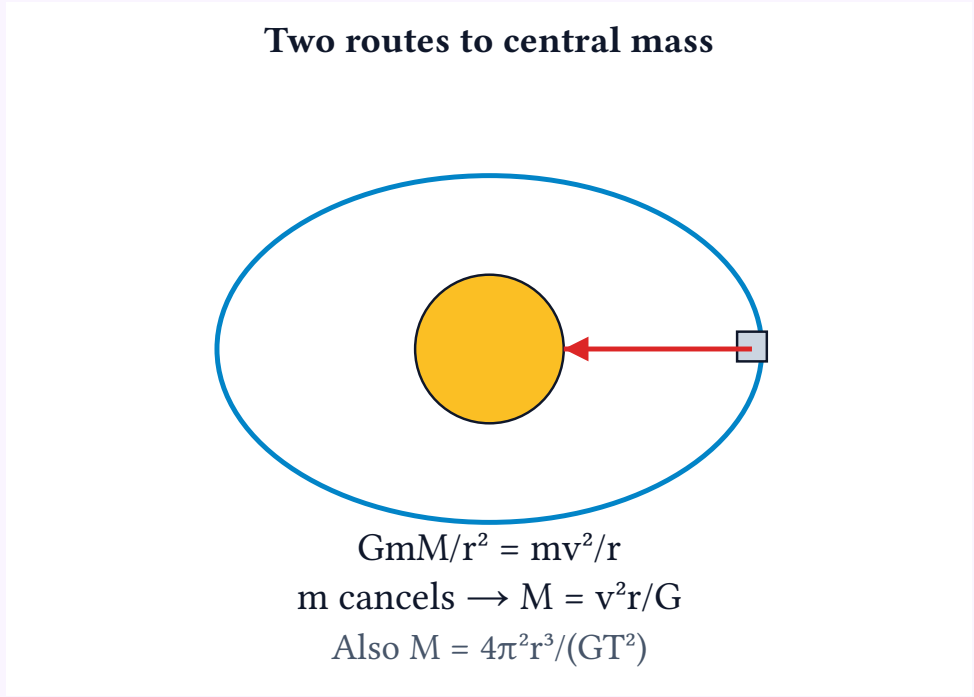
A geosynchronous orbit has a period that matches Earth's rotation period, so the satellite stays over the same longitude and can serve communication. (T = 8.64e4 s)

Kepler's laws

Kepler's laws describe orbits, and the third law says the square of the period is proportional to the cube of the average radius. (T^2 proportional to r^3)

Two routes to the central mass

- Set G M m over r squared equal to m v squared over r.
- The satellite mass m cancels, leaving $M = v^2 r / G$.
- The period form $M = 4 \pi^2 r^3 / (G T^2)$ matches that result.

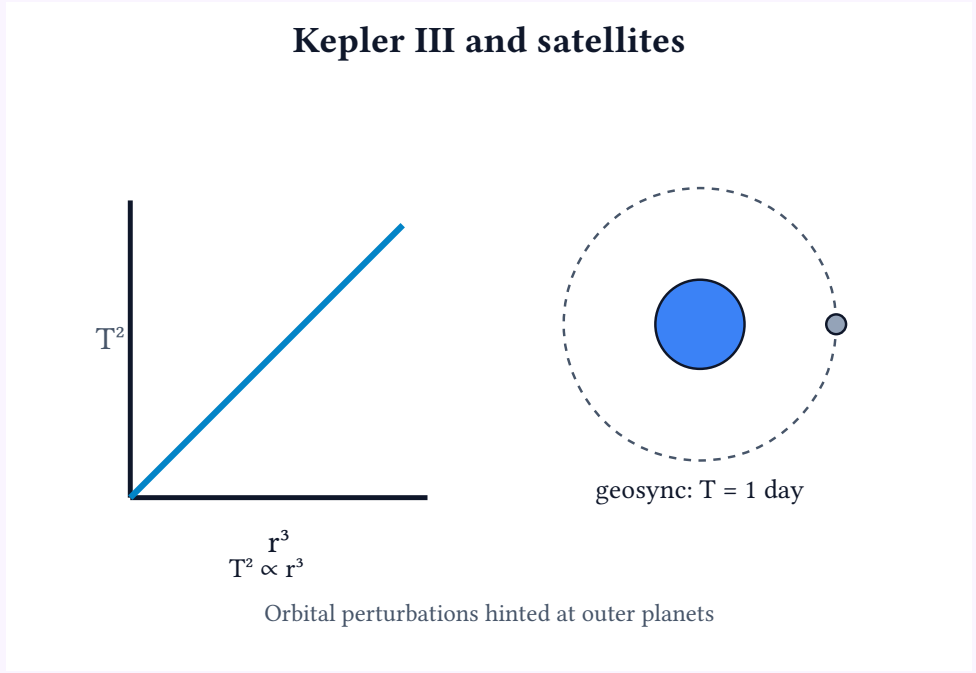


Central mass from a moonlet orbit

- Use v = 3.50 times 10 to the third m/s and r = 4.20 times 10 to the seventh metres.
- M = $(3.50\times10^{3})^{2} \times (4.20\times10^{7}) / 6.67\text{e-}11$.
- The central mass is 7.71×10^{24} kg, and the period form agrees.

Kepler’s third law and satellites

- Kepler’s third law says T squared is proportional to r cubed.
- That proportion led toward Newton’s inverse square law, and orbital perturbations later pointed to outer planets.
- A geosynchronous satellite orbits once each day, so a dish on the ground can keep pointing at it.



An outer moon’s period

- An inner moon has radius 2.00 times 10 to the eighth metres and period 2.40 times 10 to the fifth seconds.
- An outer moon of the same planet has four times the radius.
- Use Kepler’s third law to find the outer period.

Radius of a geosynchronous satellite

- Earth’s mass is 5.97 times 10 to the 24th kilograms and the rotation period used here is 8.64 times 10 to the fourth seconds.
- Find the geosynchronous radius from $M = 4\ pi^2\ r^3 / (G\ T^2)$.
- State the communication job of that orbit.

Image credits

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