Relations and functions slide deck

Mathematics 10C

Unit RF: Relations and functions

Graphs of situations

A bus ride in time

- The bus waits 2 minutes.
- It drives for 6 minutes, then waits again.
- Time keeps moving between the notes.

Inputs and outputs

- In the pair (2, 7), the input is 2 and the output is 7.
- A joined segment means the story continues between the dots.

Key terms

domain

The set of input values that the graph, table, or ordered pairs actually uses. (x-values)

range

The set of output values that correspond to those inputs. (y-values)

discrete graph

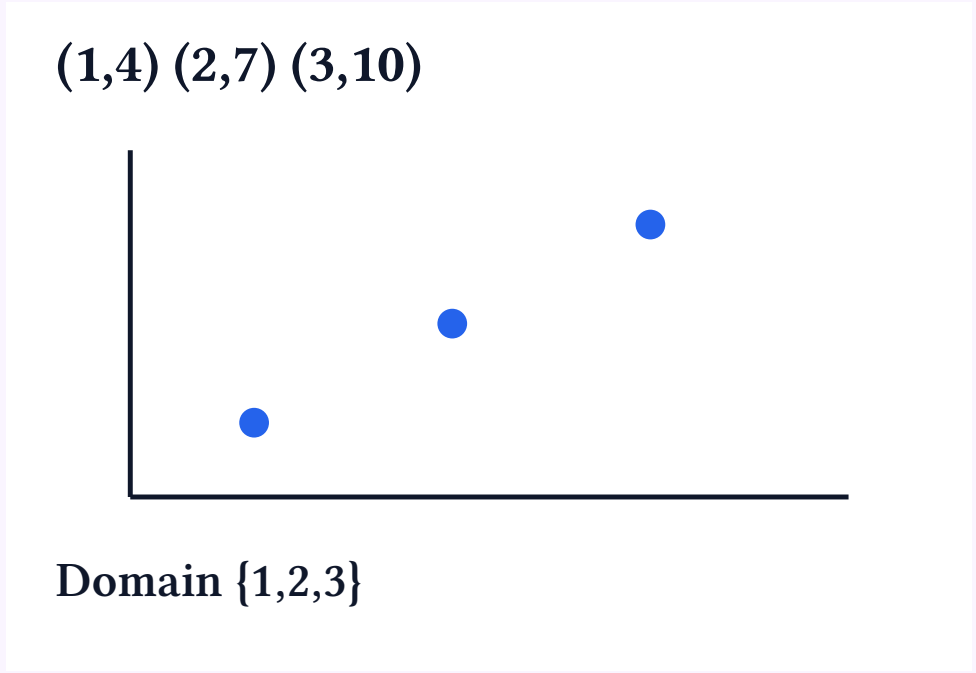
A graph of separate points, used when each point is its own count. (separate points)

continuous graph

A graph with the points joined, used when the quantity exists between the recorded values. (a joined line)

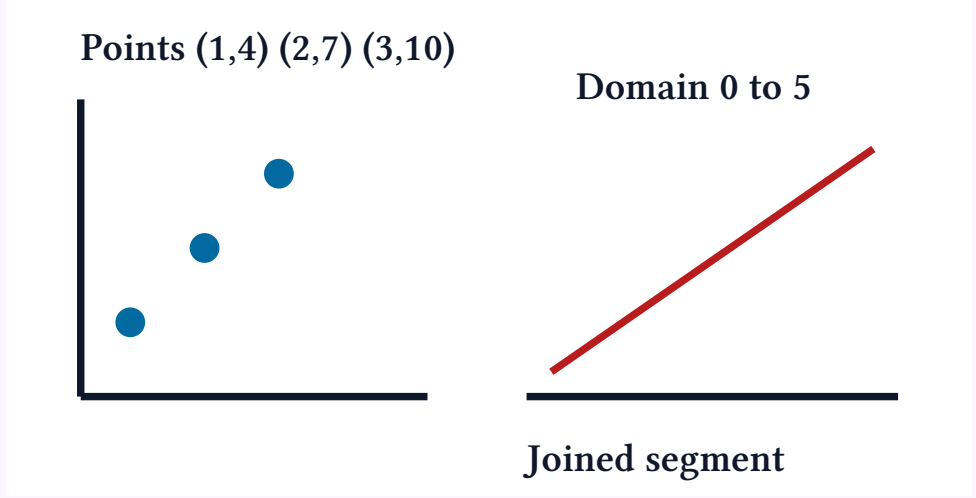
Domain and range of three points

- The pairs are (1, 4), (2, 7), and (3, 10).
- The domain is {1, 2, 3}.
- The range is {4, 7, 10}.



A segment from x = 0 to x = 5

- The segment runs from x = 0 to x = 5.
- The domain is $0 \le x \le 5$.
- If the heights run from 1 to 4, the range is $1 \le y \le 4$.

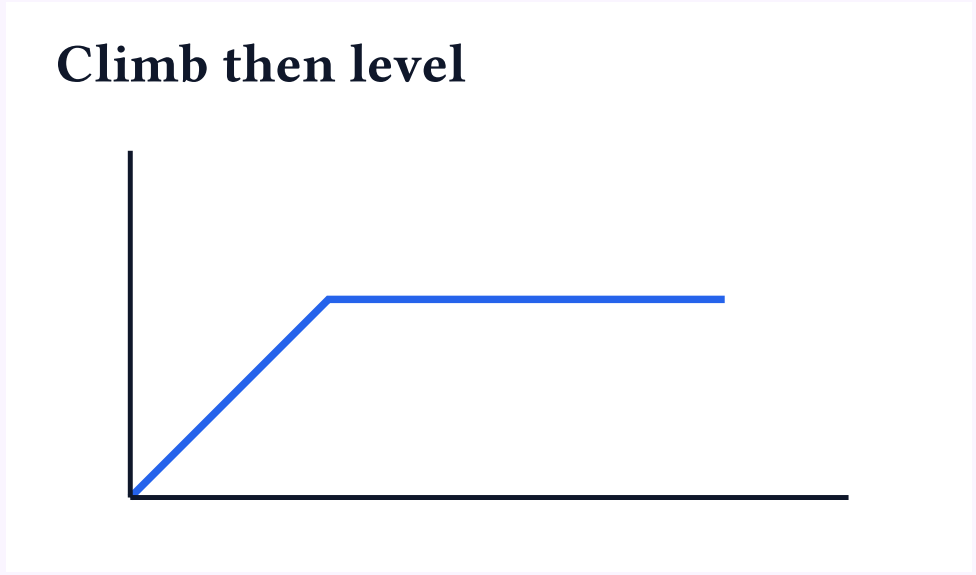


A fare table and a 12 km trip

- The table uses inputs 1, 2, and 3, so the domain is $\{1, 2, 3\}$.
- A trip from 0 km to 12 km has domain $0 \le d \le 12$.
- If the graph height runs from 0 to 12, the range is $0 \le y \le 12$.

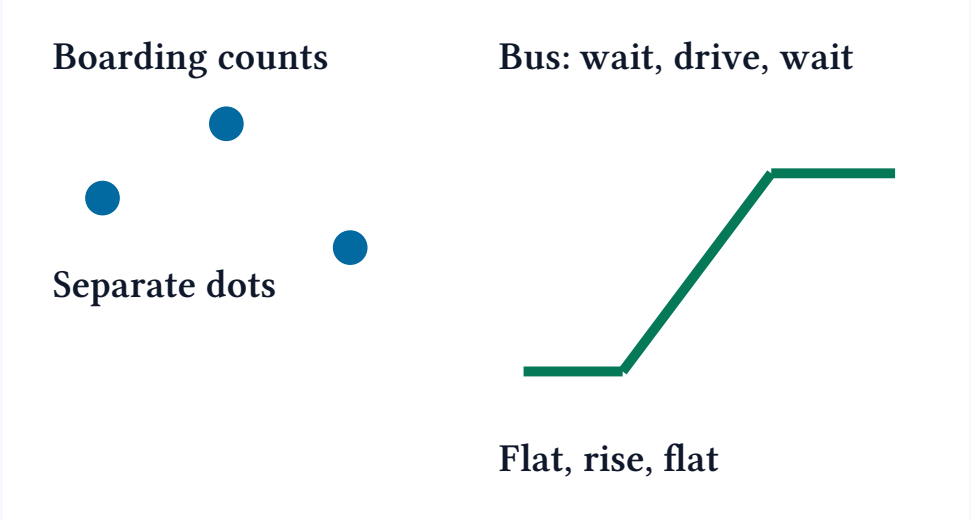
Join the points when the story continues

- From $x = 0$ to $x = 4$ the output climbs from 0 to 8.
- From $x = 4$ to $x = 7$ the output stays 8.
- The domain is $0 \le x \le 7$ and the range is $0 \le y \le 8$.



Dots for separate counts

- Riders boarding at each stop are separate counts.
- Those counts stay as dots.
- A line through the dots would invent riders between stops.



A bus trip graph

- Draw a flat piece for the first 2 minutes.
- Draw a rise for the next 6 minutes.
- Draw a flat piece for the last wait.

Quick check

- The pairs $(0, 2)$ and $(4, 2)$ have domain $\{0, 4\}$ and range $\{2\}$.
- A continuing walk between those times uses domain $0 \le t \le 4$.

Identifying a set

- Name the domain before you describe the shape.
- Join points only when the story continues between them.

Mathematics 10C

Unit RF: Relations and functions

Relations and functions

One locker, one combination

- Locker 14 has one combination.
- A sign-up lists Alex in room 4 and Alex in room 9.
- The same name has two rooms.

One output for each input

- The pairs (1, 4) and (2, 4) give input 2 a single output.
- A function pairs each input with exactly one output.

Key terms

relation

A set of ordered pairs, shown as a list, a table, or a graph. (a set of ordered pairs)

function

A relation in which each input pairs with exactly one output. (one output for each input)

ordered pair

A pair that lists an input first and its output second. ((x, y))

vertical line test

A check that slides a vertical line across a graph and expects a single meeting point. (one meeting point)

A repeated input

- The set is {(1, 4), (1, 9), (3, 7)}.
- Input 1 has outputs 4 and 9.
- This relation stays outside the functions.

Each input once

input	output
1	4
2	4
3	7

Input 1 twice

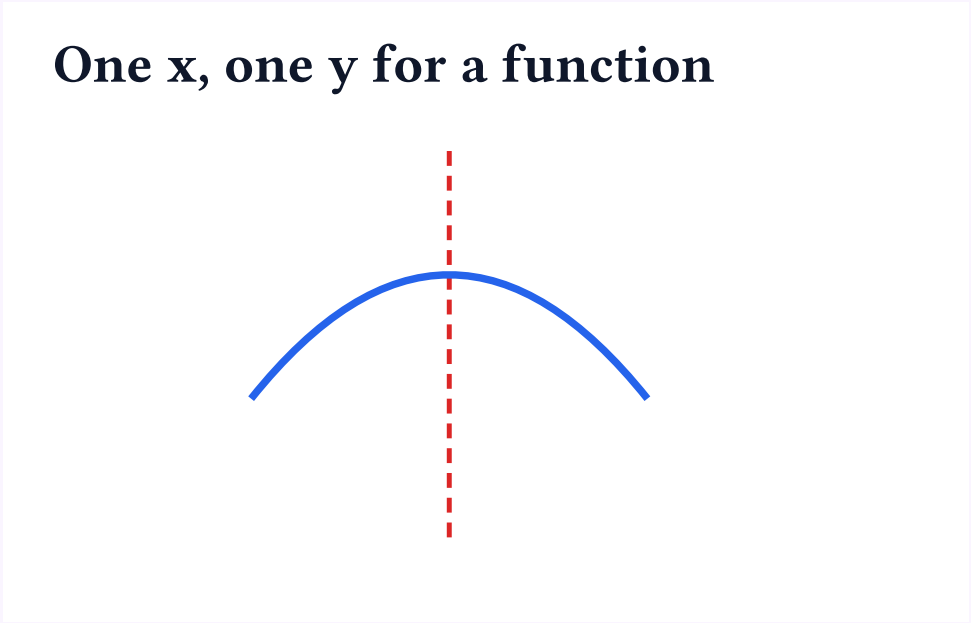
input	output
1	4
1	9
3	7

A set that passes

- The set $\{(0, 1), (2, 1)\}$ has inputs 0 and 2.
- Each input appears once.
- The set is a function, and the shared output 1 is allowed.

A vertical line on a graph

- A vertical line through one x meets a function graph once.
- A graph shaped like a C opening to the right meets that line twice.
- The C-shaped graph stays outside the functions.



Rules in words

- Each locker number pairs with one combination.
- That sentence is the function rule, with the graph put away.

One locker → one combo

Function rule in words

Four examples

- Set A = {(0, 1), (2, 1)} is a function.
- Set B = {(3, 4), (3, 5)} stays outside the functions.
- A horizontal line is a function. A C opening right stays outside.

Quick check

- The set {(2, 5), (2, 8), (4, 5)} stays outside the functions.
- A vertical line meets a horizontal graph once, so that graph is a function.

One output per input

- Check the inputs before you check the outputs.
- A repeated input with two outputs stays outside the functions.

Mathematics 10C

Unit RF: Relations and functions

Slope of a line

A ramp and a staircase

- The ramp rises 1 m over a 4 m run.
- The staircase rises 1 m over a 1 m run.
- The staircase is steeper.

Rise over run

- Slope is rise divided by run.
- A line through (1, 2) and (4, 8) has slope 2.

Key terms

slope

The ratio of vertical change to horizontal change along a line. (m = rise/run)

rise

The vertical change between two points, positive when the line goes up. (change in y)

run

The horizontal change between two points, positive when moving to the right. (change in x)

rate of change

A slope that tells how much one quantity changes for each unit of another. (dollars per kilometre)

Key terms

parallel lines

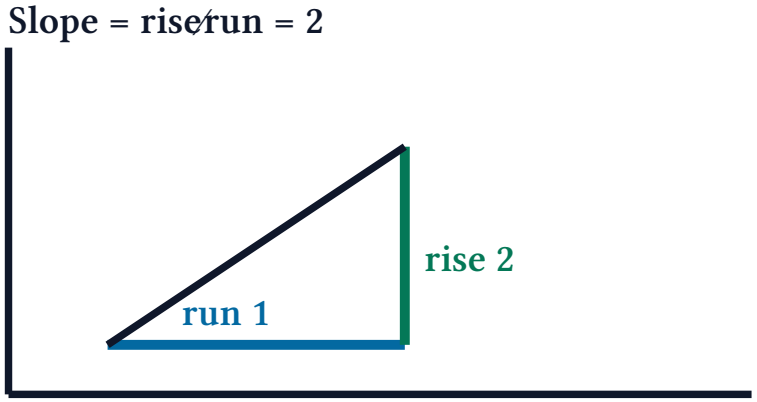
Lines that share one slope and stay the same distance apart forever. (equal slopes)

perpendicular lines

Lines that meet at a right angle. For two sloping lines, the product of the slopes is −1. (slopes multiply to −1)

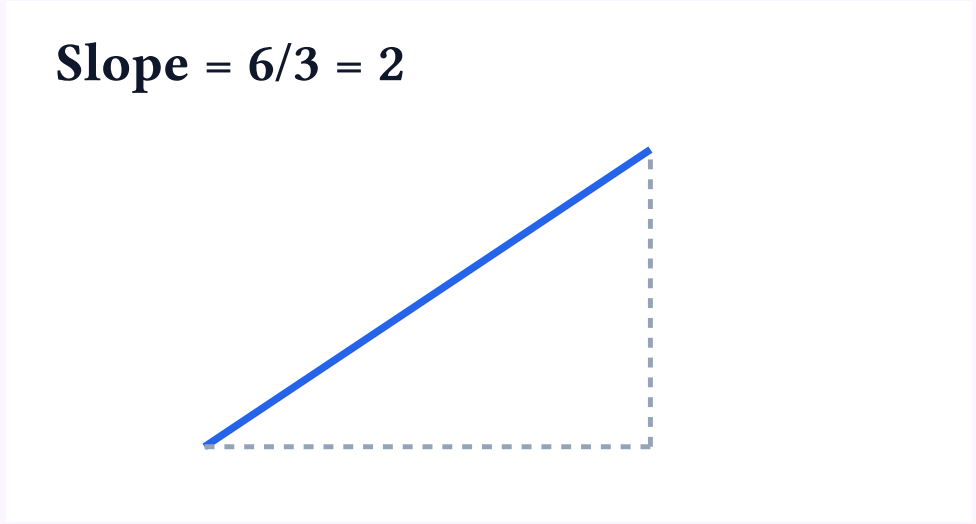
Slope from (1, 2) to (4, 8)

- The rise is $8 - 2 = 6$.
- The run is $4 - 1 = 3$.
- The slope is $\frac{6}{3} = 2$.



The same slope farther along

- From (4, 8) to (7, 14), the rise is 6.
- The run is 3.
- The slope is again 2.

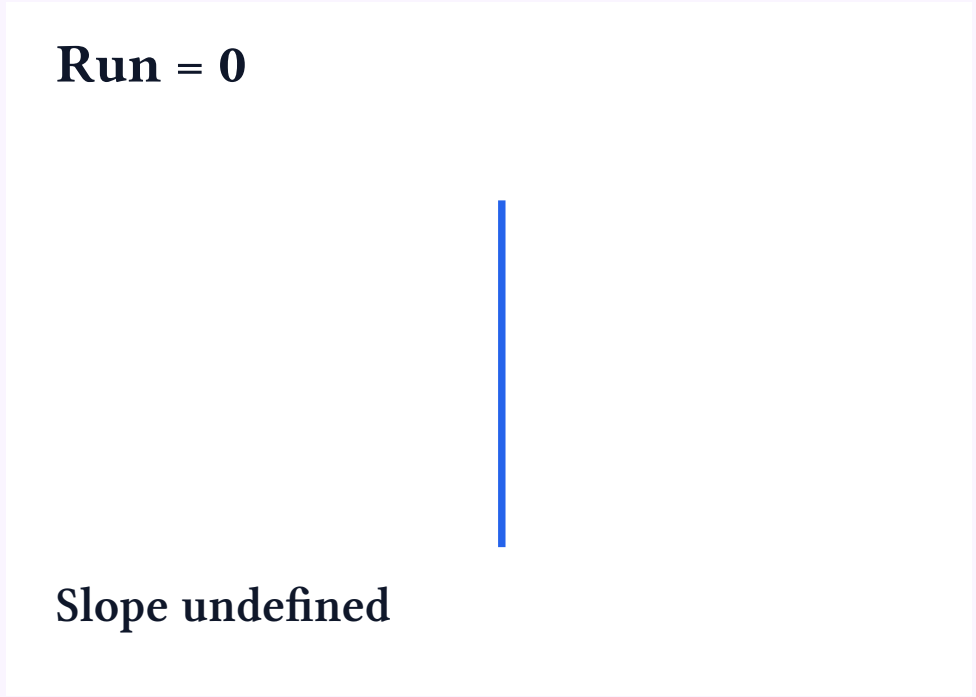


A falling line and a flat line

- From $(0, 5)$ to $(2, 1)$, the rise is $1 - 5 = -4$.
- The run is 2, so the slope is -2.
- A horizontal line from $(0, 2)$ to $(6, 2)$ has rise 0 and slope 0.

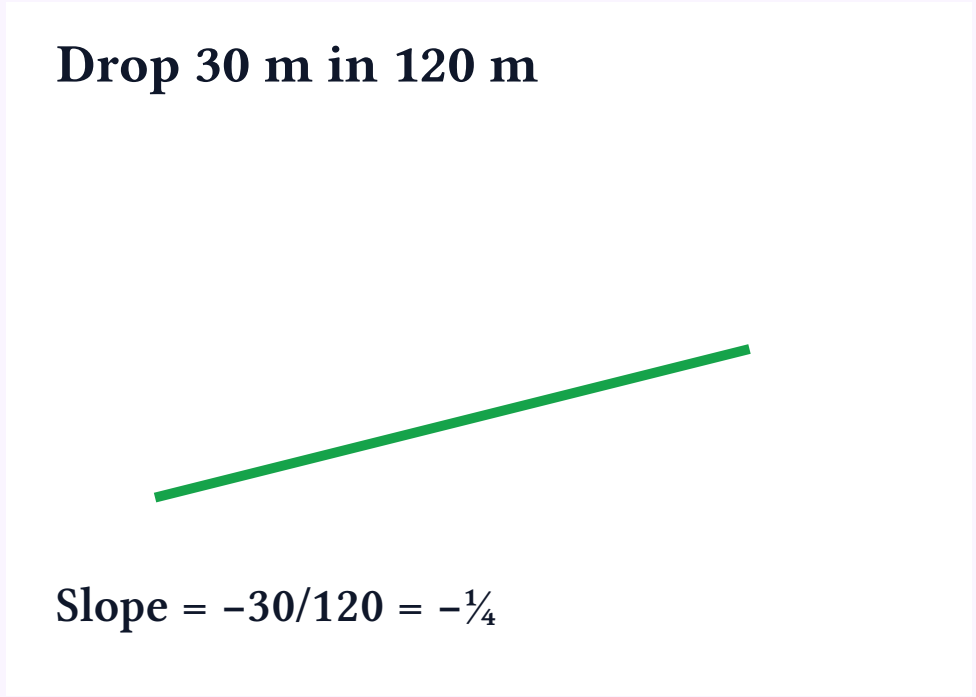
A vertical line

- The points (3, 4) and (3, 9) share an x-coordinate.
- The run is $3 - 3 = 0$.
- The slope is undefined, and the line is vertical.



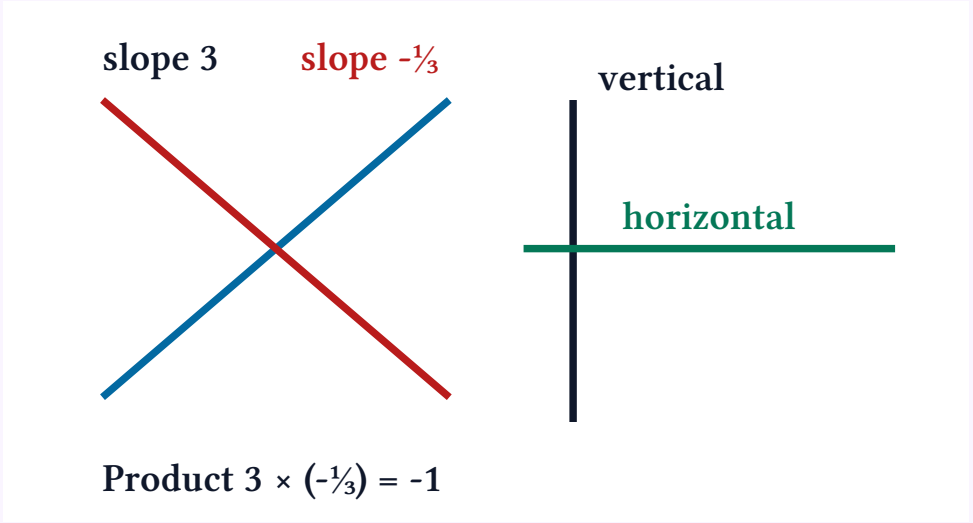
A trail as a rate

- The trail drops 30 m over a run of 120 m.
- The slope is $-\frac{30}{120} = -\frac{1}{4}$.
- That rate is 1 m of drop for every 4 m of run.



A new point and a right angle

- From (2, 1) with slope 3, one run and three rises land on (3, 4).
- A parallel line also has slope 3.
- A perpendicular slope is $-\frac{1}{3}$, because $3 \times (-\frac{1}{3}) = -1$.



A hill of rise 15

- The rise is 15 m and the run is 45 m.
- The slope is $^{15}\!/_{45} = \frac{1}{3}$.

Quick check

- A drop of 8 m over a run of 24 m has slope $-\frac{8}{24} = -\frac{1}{3}$.
- The slope perpendicular to 2 is $-\frac{1}{2}$, because $2 \times \left(-\frac{1}{2}\right) = -1$.

Slope as a ratio

- Slope 2 means rise 2 for run 1.
- A run of 0 leaves the slope undefined.

Mathematics 10C

Unit RF: Relations and functions

Representing linear relations

A taxi fare in two lines

- The flag drop is $4.
- Each kilometre adds $2.
- At 0 km the fare is $4, and at 1 km it is $6.

Five versions of one fare

- The equation is C = 2d + 4.
- The graph through (0, 4) and (1, 6) is a straight line.

Key terms

linear relation

A relation whose rate of change is constant and whose graph is a straight line.
($y = 2x + 4$)

independent variable

The input you choose, written in the first coordinate or on the horizontal axis. (x or d)

dependent variable

The output that responds to the input, written in the second coordinate or on the vertical axis. (y or C)

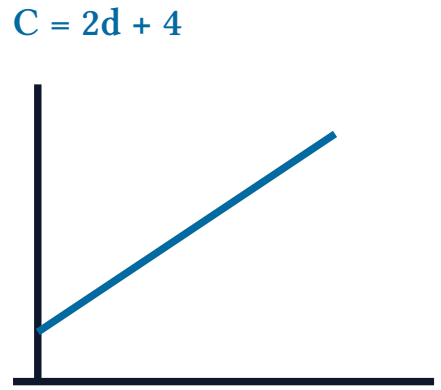
first difference

The change in consecutive outputs when the inputs increase by the same step. (Δy)

Three representations

- The fares 4, 6, 8, 10 rise by 2 each kilometre.
- Those equal jumps make a straight line.
- The equation is C = 2d + 4.

km	cost
0	$4
1	$6
2	$8



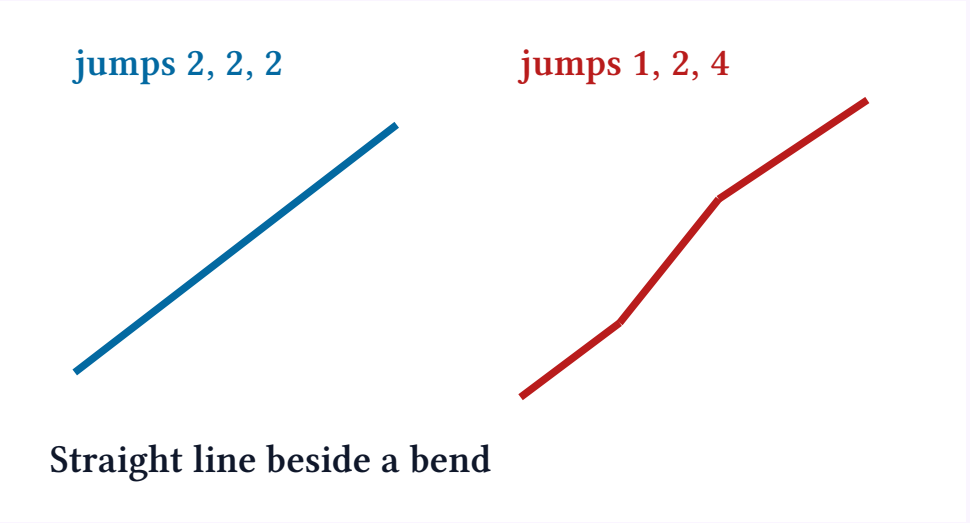
Table, line, and equation agree

Testing the point (3, 10)

- In the story, 3 km costs $4 + 2 \times 3 = 10$ dollars.
- The pair (3, 10) is on the line.
- The equation $C = 2(3) + 4$ also gives 10.

Equal jumps or changing jumps

- The outputs 1, 3, 5, 7 have differences 2, 2, and 2, so the relation is linear.
- The outputs 1, 2, 4, 8 have differences 1, then 2, then 4, so the relation is nonlinear.



Four representations

- A curve is nonlinear.
- The equation $y = -x + 6$ is linear.
- Outputs 10, 7, 4, 1 have differences −3, −3, and −3, so that table is linear.
- The area of a square as the side grows is nonlinear.

Quick check

- The pair (2, 8) fits C = 2d + 4, because 2(2) + 4 = 8.
- Outputs 2, 5, 10 are nonlinear because the jumps are 3 and 5.

Displays that agree

- The fare $C = 2d + 4$ matches the points $(0, 4)$ and $(1, 6)$.
- A changing jump means the graph bends.

Mathematics 10C

Unit RF: Relations and functions

Characteristics of linear graphs

Where the line meets the axes

- The line passes through (0, 4) and (2, 0).
- It meets the y-axis at 4 and the x-axis at 2.

Features of a linear graph

- The slope between $(0, 4)$ and $(2, 0)$ is $(0 - 4)/(2 - 0) = -2$.
- A full line uses every real number as an input and as an output.

Key terms

x-intercept

The point where a graph meets the horizontal axis, so the output is 0. ((a, 0))

y-intercept

The point where a graph meets the vertical axis, so the input is 0. ((0, b))

domain

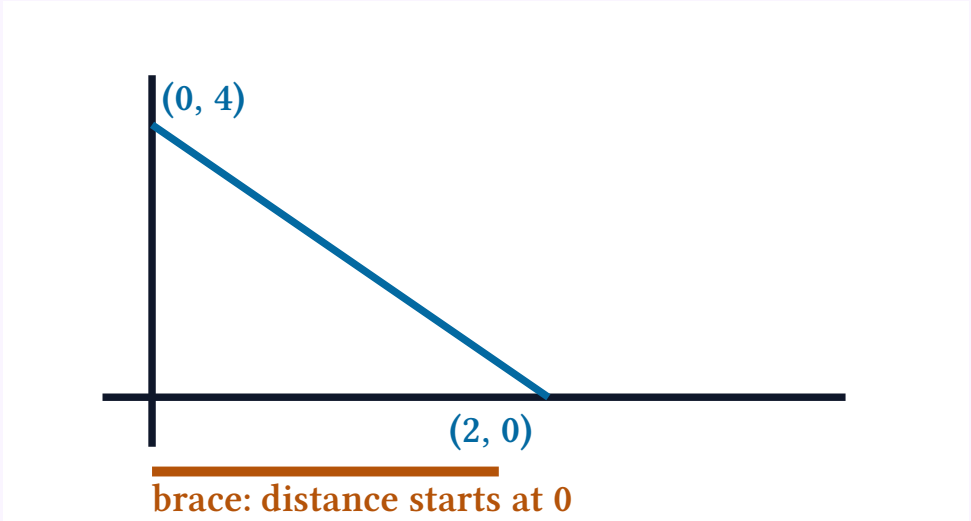
The set of inputs the relation uses, all real numbers for a full non-vertical line. (allowed x)

range

The set of outputs the relation uses, all real numbers for a full non-horizontal line. (allowed y)

Four features of the full line

- The y-intercept is (0, 4).
- The x-intercept is (2, 0).
- The slope is −2, and the domain and range are all real numbers.

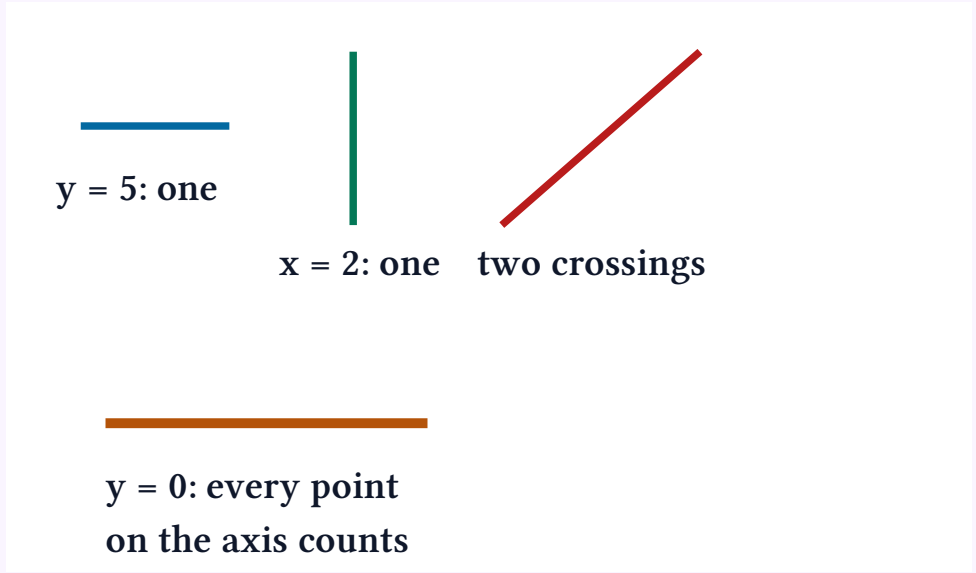


A phone bill

- The bill is $C = 0.10m + 20$ for $m \geq 0$.
- The y-intercept is $(0, 20)$ and the slope is 0.10.
- The domain is $m \geq 0$ and the range is $C \geq 20$.

How many intercepts

- The line $y = x + 1$ meets the axes at $(0, 1)$ and $(-1, 0)$.
- The horizontal line $y = 3$ meets the y-axis once, at $(0, 3)$.
- The x-axis, $y = 0$, has every point on it as an x-intercept.



A ray of the fare

- With $x \geq 0$ and $y \geq 0$, the graph runs from $(0, 3)$ to $(3, 0)$.
- The domain is $0 \leq x \leq 3$ and the range is $0 \leq y \leq 3$.

Quick check

- The line through $(0, -1)$ and $(2, 3)$ has slope 2 and y-intercept -1.
- The bill $C = 0.10m + 20$ with $m \geq 0$ has range $C \geq 20$.

Reading four features

- Name the intercepts, then the slope.
- Then say whether a story cuts the domain or the range.

Mathematics 10C

Unit RF: Relations and functions

Forms of a line

One line, three equations

- The equation y = $-3x + 6$ has slope −3 and y-intercept 6.
- The equation $3x + y - 6 = 0$ rearranges to that same line.
- The equation y − 6 = −3(x − 0) is the same line again.

Three forms

- Slope-intercept form shows m and b, as in $y = -3x + 6$.
- General form sets one side to 0, as in $3x + y - 6 = 0$.
- Slope-point form starts from a known point, as in $y - 6 = -3(x - 0)$.

Key terms

slope-intercept form

A linear equation that shows the slope m and the y-intercept b. (y = mx + b)

general form

A linear equation written as a sum of terms equal to 0. (Ax + By + C = 0)

slope-point form

A linear equation built from the slope and one point on the line. (y - y1 = m(x - x1))

Two slope-point versions

- Using $(0, 6)$: $y - 6 = -3(x - 0)$.
- Using $(2, 0)$: $y - 0 = -3(x - 2)$.
- Both simplify to $y = -3x + 6$.

Slope −3

$y - 6 = -3(x - 0)$

$y = -3(x - 2)$

From general form to slope and intercept

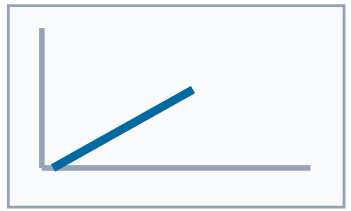
- Start with $2x - 4y + 8 = 0$.
- Then $4y = 2x + 8$, so $y = (\frac{1}{2})x + 2$.
- The slope is $\frac{1}{2}$ and the y-intercept is 2.

Graphing from any form

- For $y = (\frac{1}{2})x + 2$, start at (0, 2) and rise 1 across a run of 2.
- For $2x - 4y + 8 = 0$, the intercepts are (−4, 0) and (0, 2).
- For $y - 2 = (\frac{1}{2})(x - 4)$, plot (4, 2) and use the slope.



Stair
(0, 2), rise 1 run 2



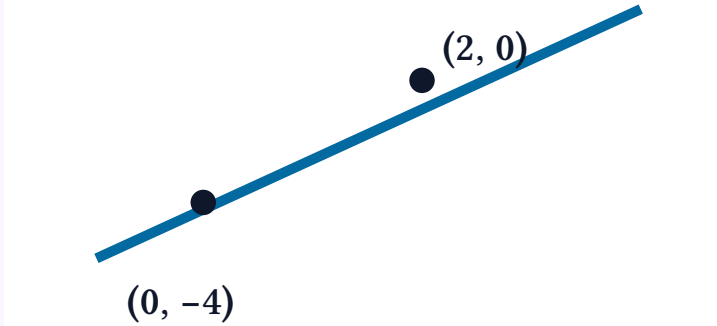
Intercepts
(−4, 0) and (0, 2)



Point
(4, 2), slope ½

Matching three equations

- The set is $y = 2x - 4$.
- The set includes $x - (\frac{1}{2})y - 2 = 0$.
- The set includes $y - 0 = 2(x - 2)$.
- All three simplify to $y = 2x - 4$.



$y = 2x - 4$

$x - \frac{1}{2}y - 2 = 0$

$y = 2(x - 2)$

Rewriting $y - 1 = 4(x - 2)$

- Expand to $y - 1 = 4x - 8$.
- Then $y = 4x - 7$.
- The slope is 4 and the y-intercept is -7.

Quick check

- The equation $3x + y - 6 = 0$ rewrites as y = $-3x + 6$.
- The graph starts at (0, 6) and falls 3 for a run of 1.

Choosing a form

- Use slope-intercept form when you want m and b.
- Use slope-point form when you already know a point and a slope.

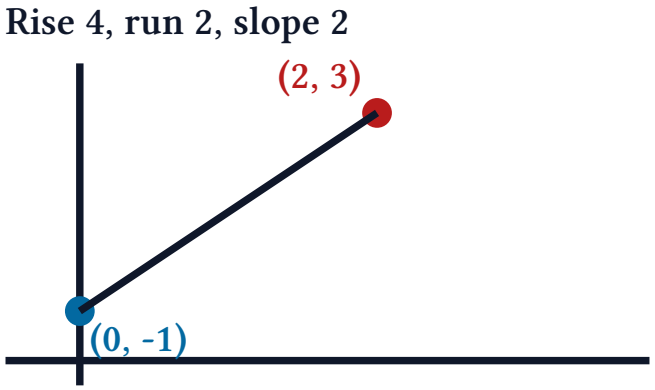
Mathematics 10C

Unit RF: Relations and functions

Equations of a line

Equation from a graph

- The line passes through (0, −1) and (2, 3).
- The rise is 4 and the run is 2, so the slope is 2.
- The equation is $y = 2x - 1$.



Writing an equation

- From slope 2 and the point (1, 4), write $y - 4 = 2(x - 1)$.
- A parallel line keeps that slope. A perpendicular line uses the slope that multiplies with it to -1.

Key terms

slope-intercept equation

The equation of a line written so the slope and the y-intercept are the two numbers you can see. (y = mx + b)

two-point slope

The slope you get by subtracting the coordinates of two points on the line. (m = (y2 - y1)/(x2 - x1))

parallel slope

The slope you copy from a line when the new line must run in the same direction. (same m)

perpendicular slope

The slope that multiplies the given slope to make −1, so the lines meet at a right angle. (m1 times m2 = -1)

Two points on a pay line

- The pay is $50 at 2 hours and $90 at 6 hours.
- The slope is $(90 - 50)/(6 - 2) = 10$ dollars per hour.
- Using $(2, 50)$: $y - 50 = 10(x - 2)$, so $y = 10x + 30$.

Checking a pay equation

- At x = 2, y = 10 × 2 + 30 = 50.
- At x = 6, y = 10 × 6 + 30 = 90.
- At x = 0, the pay is $30.

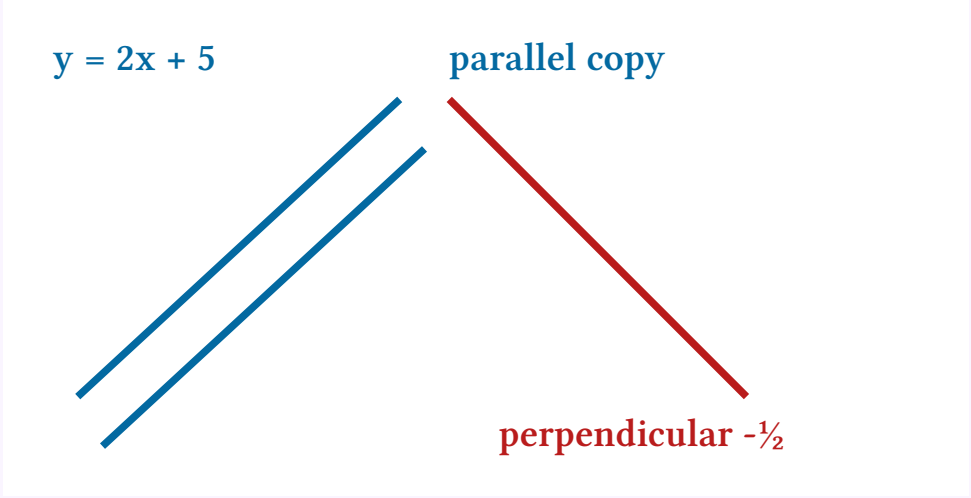
y = 10x + 30

x=2 ⟶ y=50

x=6 ⟶ y=90

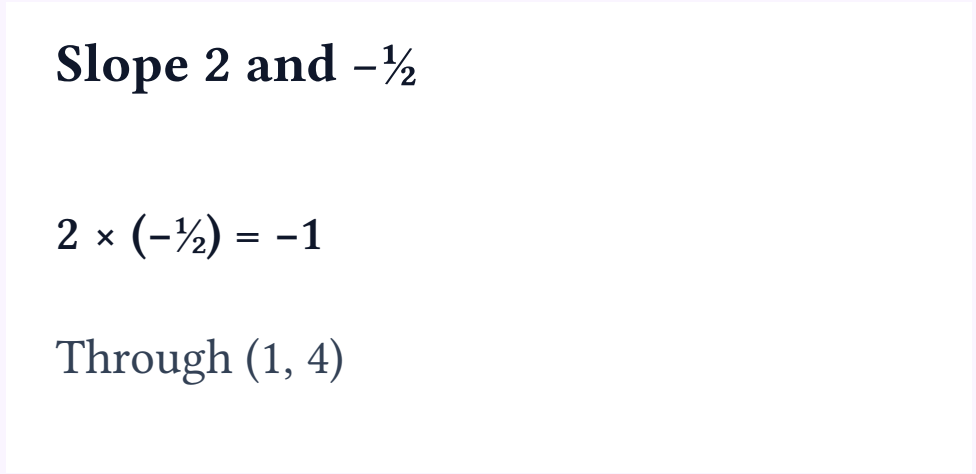
A parallel line through (1, 4)

- The line $y = 2x + 5$ has slope 2.
- The parallel line through (1, 4) keeps slope 2.
- The equation $y - 4 = 2(x - 1)$ simplifies to $y = 2x + 2$.



A perpendicular line through (1, 4)

- The slope perpendicular to 2 is $-\frac{1}{2}$.
- Check: $2 \times (-\frac{1}{2}) = -1$.
- The equation is $y - 4 = (-\frac{1}{2})(x - 1)$.



Writing and testing

- The line through (0, 1) and (3, 7) has slope 2.
- The equation is $y = 2x + 1$.
- At $x = 3$, $y = 7$.

Quick check

- A line through $(0, -1)$ with slope 4 has equation $y = 4x - 1$.
- A line parallel to it through $(2, 3)$ has equation $y - 3 = 4(x - 2)$.

Testing an equation

- Build the slope, then anchor a point.
- Substitute a known point as a check.

Mathematics 10C

Unit RF: Relations and functions

Function notation

Substituting −1

- The rule is f(x) = $3x - 2$.
- The input −1 gives f(−1) = 3(−1) − 2 = −5.

A name for the output

- The rule f(x) = $3x$ — 2 matches y = 3x − 2.
- The symbol g(t) = 7 + t uses t as the input name.

Key terms

function notation

A way to name a function and its output, with the input written in parentheses. (f(x) = $3x - 2$)

domain value

A number you are allowed to put into the function. (the input x)

range value

The number the function gives back for a domain value. (the output f(x))

From an input to an output

- The input -1 goes into $f(x) = 3x - 2$.
- The calculation is $-3 - 2$.
- The output is -5, so the point is $(-1, -5)$.



f(x) = 3x - 2

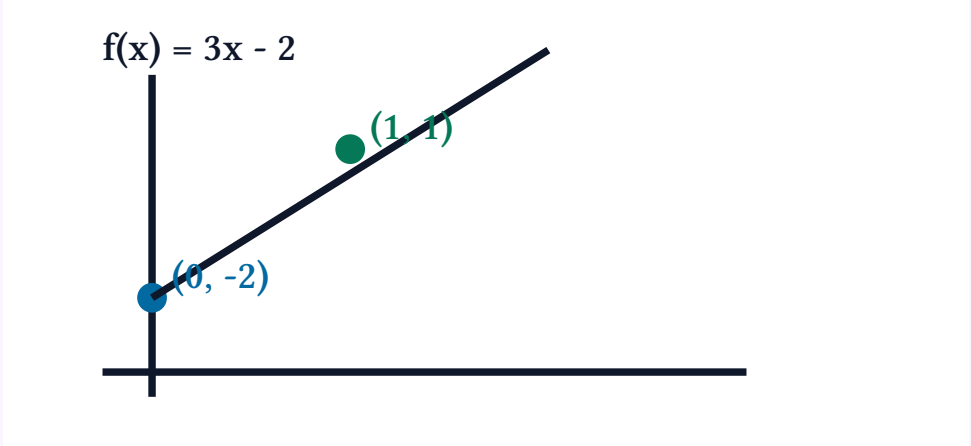
Point on the line: (-1, -5)

From an output back to the input

- Solve $g(t) = 15$ when $g(t) = 7 + t$.
- The equation $7 + t = 15$ gives $t = 8$.
- The input 8 produces the output 15.

Sketching f(x) = $3x - 2$

- The y-intercept is (0, −2), because f(0) = −2.
- The slope is 3, so another point is (1, 1).
- Draw the line through (0, −2) and (1, 1).



Using function notation

- The value $h(2)$ for $h(x) = 5 - 2x$ is $5 - 4 = 1$.
- Solving $h(x) = 1$ gives $5 - 2x = 1$, so $x = 2$.
- In two variables the rule is $y = 5 - 2x$.

Quick check

- The value f(4) for f(x) = $3x - 2$ is 10.
- Solving g(t) = 15 for g(t) = 7 + t gives t = 8.

Evaluating a function

- The value $f(-1) = -5$ for $f(x) = 3x - 2$.
- The graph of f is the line $y = 3x - 2$.

Mathematics 10C

Unit RF: Relations and functions

Systems of linear equations

Two lines that meet

- The lines are $y = x + 1$ and $y = -x + 5$.
- At $x = 2$ both give $y = 3$.
- The shared point $(2, 3)$ is the solution of the system.

A pair that works twice

- A system here is two linear equations.
- The solution of a system is the pair that makes both equations true.

Key terms

system of linear equations

Two linear equations considered together, each drawing a line. (two equations)

solution of a system

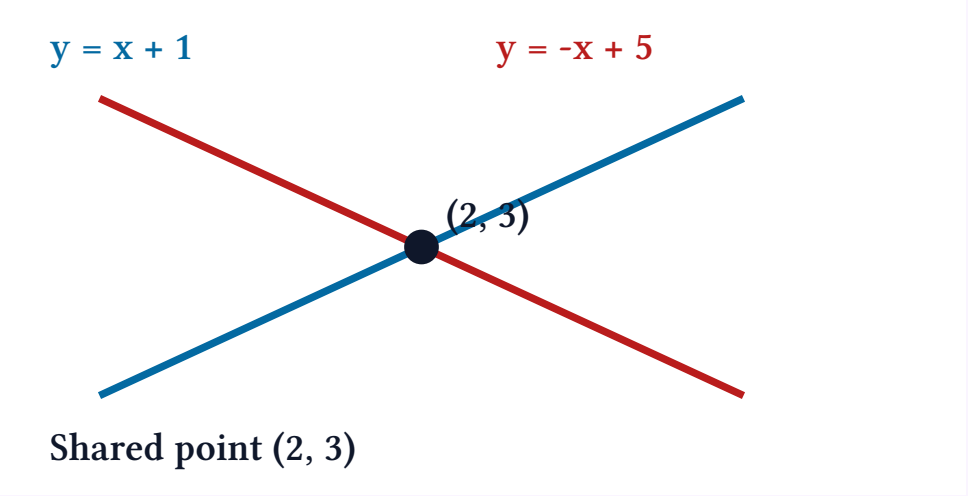
The ordered pair that makes both equations true at the same time. ((x, y))

point of intersection

The point where the two lines meet, which is the solution you can see on the graph. (the shared point)

The intersection point

- Set x + 1 equal to −x + 5.
- Then $2x = 4$, so x = 2 and y = 3.
- Check: 2 + 1 = 3 and −2 + 5 = 3.

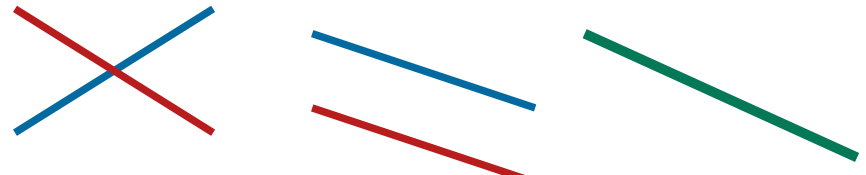


Add to remove y

- The equations are x + y = 5 and x − y = 1.
- Adding them gives $2x = 6$, so x = 3.
- Then y = 2, and the checks are 3 + 2 = 5 and 3 − 2 = 1.

How many solutions

- The lines $y = 2x + 1$ and $y = -x + 4$ meet once.
- The lines $y = 2x + 1$ and $y = 2x + 4$ stay apart.
- The equation $2y = 4x + 2$ is the line $y = 2x + 1$ again.



one point lines apart one shared line

Different slopes meet once.

Same slope, new intercept: apart.

One intersection by algebra

- Set $2x + 1$ equal to −x + 4.
- Then $3x = 3$, so x = 1 and y = 3.
- Check: 2(1) + 1 = 3 and −1 + 4 = 3.

2x + 1 = −x + 4

3x = 3 ⟶ x = 1

y = 3

Two phone plans

- Plan A is $y = 0.10x + 20$.
- Plan B is $y = 0.20x + 10$.
- Set them equal: $0.10x + 20 = 0.20x + 10$, so $x = 100$ and $y = 30$.

Quick check

- Solving y = x + 4 and y = $-2x + 10$ gives (2, 6).
- The lines y = 3x + 1 and y = 3x − 5 stay apart.

A pair that works twice

- The lines $y = x + 1$ and $y = -x + 5$ meet at $(2, 3)$.
- A solution makes both equations true.