Mathematics 10C

Unit M: Measurement and trigonometry

Surface area and volume

A grain bin and its roof

- The bin is a cylinder.
- The roof is a cone on the same radius.
- Grain fills the space. Paint covers the outside.



Five solids, two measures

- Volume of a cylinder is the base area times the height.
- Volume of a matching cone is one third of that product.

Key terms

surface area

The total area of the exterior faces of a solid, measured in square units. (SA)

volume

The amount of space a solid occupies, measured in cubic units. (V)

right prism

A prism whose sides stand perpendicular to the bases, so the height meets the base at a right angle. ($V = Bh$)

right cylinder

A solid with two parallel circular bases and a curved side perpendicular to those bases. ($V = \pi r^2 h$)

Key terms

right pyramid

A solid with a polygon base and triangular faces that meet at an apex above the centre of the base. ($V = (\frac{1}{3})Bh$)

right cone

A solid with a circular base and a smooth side that meets at an apex above the centre of the base. ($V = (\frac{1}{3})\pi r^2 h$)

sphere

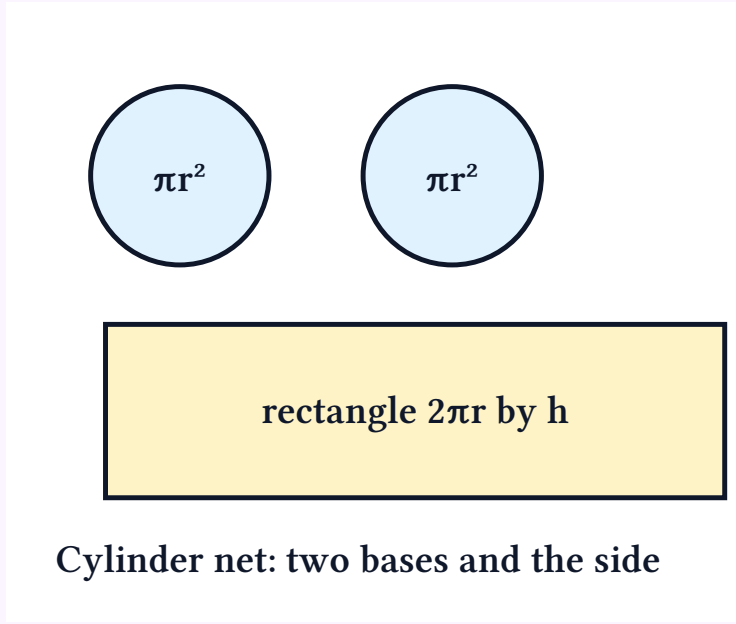
The set of points in space at a fixed distance, the radius, from a centre. ($V = (\frac{4}{3})\pi r^3$)

slant height

The distance along the sloping face from the base edge to the apex, used in cone and pyramid surface area. (s)

Cylinder bases

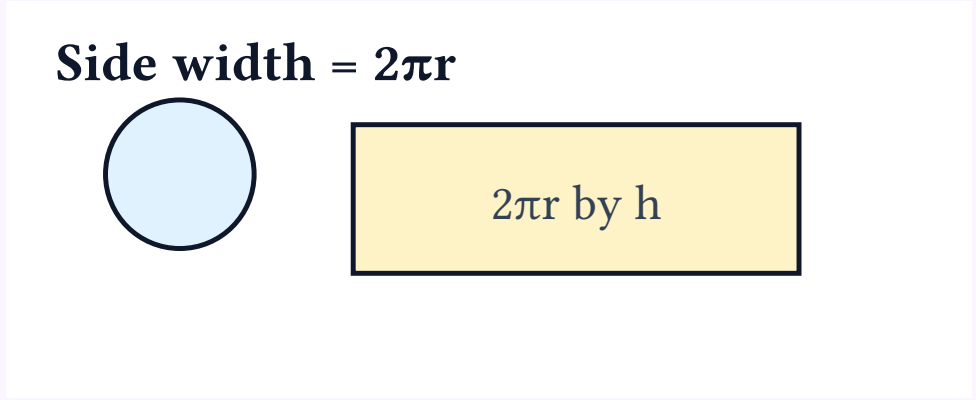
- The radius is 3 cm and $\pi = 3.14$.
- One base is $3.14 \times 9 = 28.26$ cm^2.
- Two bases are 56.52 cm^2.



Cylinder net: two bases and the side

Cylinder side

- The side is a rectangle.
- Its width is the circumference
  $2 \times 3.14 \times 3$.
- Its area is
  $18.84 \times 10 = 188.4$ cm^2.

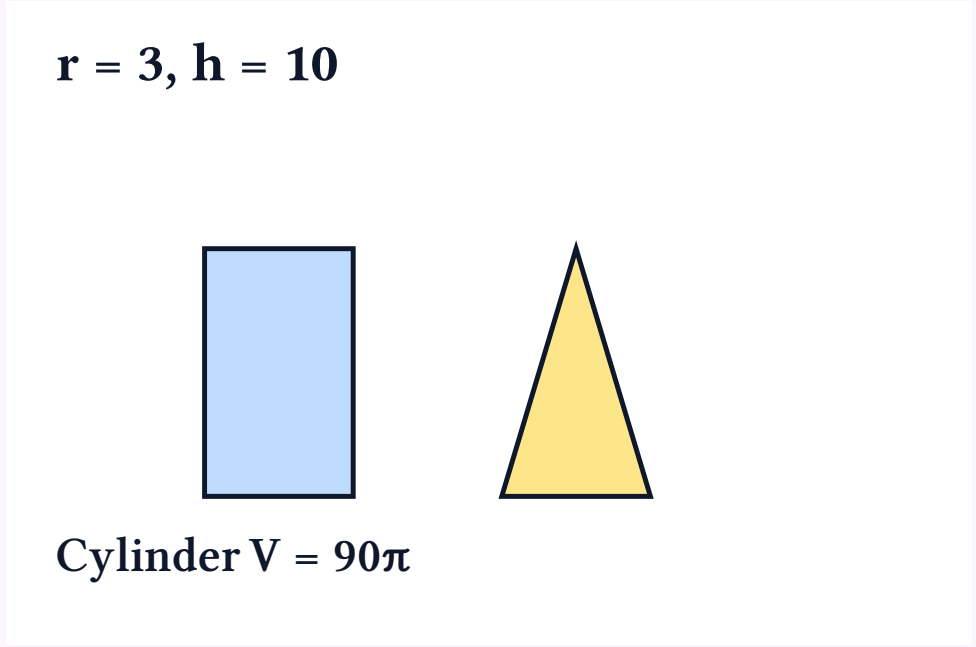


Adding the cylinder surface

- The bases contribute 56.52 cm^2.
- The side contributes 188.4 cm^2.
- The surface area is 244.92 cm^2.

Cone beside a matching cylinder

- Both solids use r = 3 and h = 10.
- The cylinder volume is 90π.
- The cone volume is 30π.



A missing cylinder height

- The volume is 100π and the radius is 5.
- The base area is 25π.
- The height is $100\pi / 25\pi = 4$.

$V = 100\pi$, $r = 5$

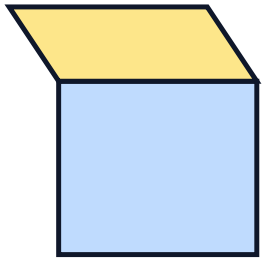
Base area $\pi r^2 = 25\pi$

$h = V \div \text{base} = 4$

Bin volume

- The cylinder part is $\pi \times 4 \times 5 = 20\pi$.
- The cone roof is one third $\times \pi \times 4 \times 3 = 4\pi$.
- The total volume is 24π.

Cylinder + cone roof



$20\pi + 4\pi = 24\pi$

Exterior paint on the bin

- The bottom is 4π.
- The cylinder side is 20π.
- The cone side is $2\pi\sqrt{13}$. The shared circle stays inside.

Quick check

- The radius is 3 cm and $\pi = 3.14$.
- The surface area is $4 \times 3.14 \times 9 = 113.04$ cm^2.
- The volume is $(\frac{4}{3}) \times 3.14 \times 27 = 113.04$ cm^3.

A formula needs matching units

- Write the known radius and height first.
- Then choose volume or surface area.

Image credits

Corey Coyle, Harvestore® Silo and Grain Bins - panoramio, CC BY 3.0