Measurement and trigonometry slide deck

Mathematics 10C

Unit M: Measurement and trigonometry

Linear measurement with referents and tools

A doorway and a thumb

- A doorway is about 1 m across.
- A thumb is about 1 inch across.
- A couch wider than that doorway needs another path.

Unit referents

- A long stride stands for about 1 m.
- A size-ten shoe stands for about 1 foot.

Key terms

referent

A familiar object or body length that stands in for a unit before a measuring tool is used. (a familiar length)

SI unit

A length unit in the International System, built by powers of ten from the metre. (mm, cm, m, km)

imperial unit

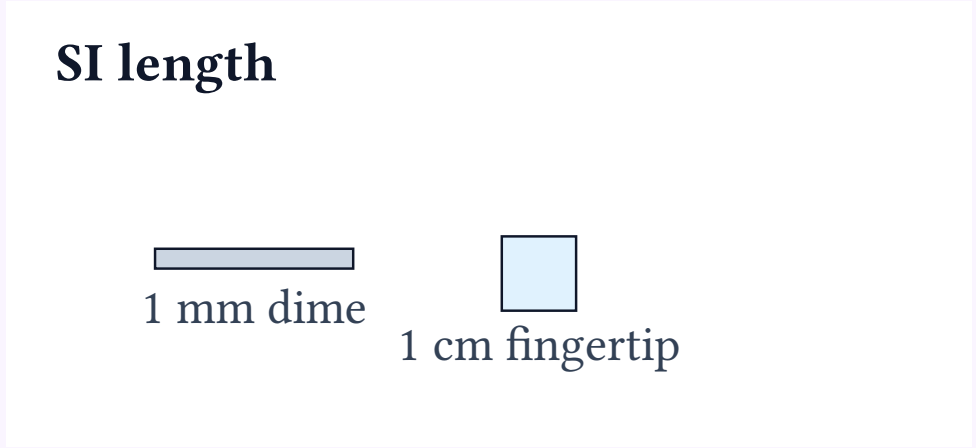
A length unit in the inch-foot-yard-mile system still used on many North American tools and signs. (in, ft, yd, mi)

linear measurement

The size of a distance along a line, a curve, or the edge of an object, reported with a unit. (a one-dimensional length)

SI units of length

- A dime is about 1 mm thick.
- A fingertip is about 1 cm across.
- A brisk 12-minute walk covers about 1 km.



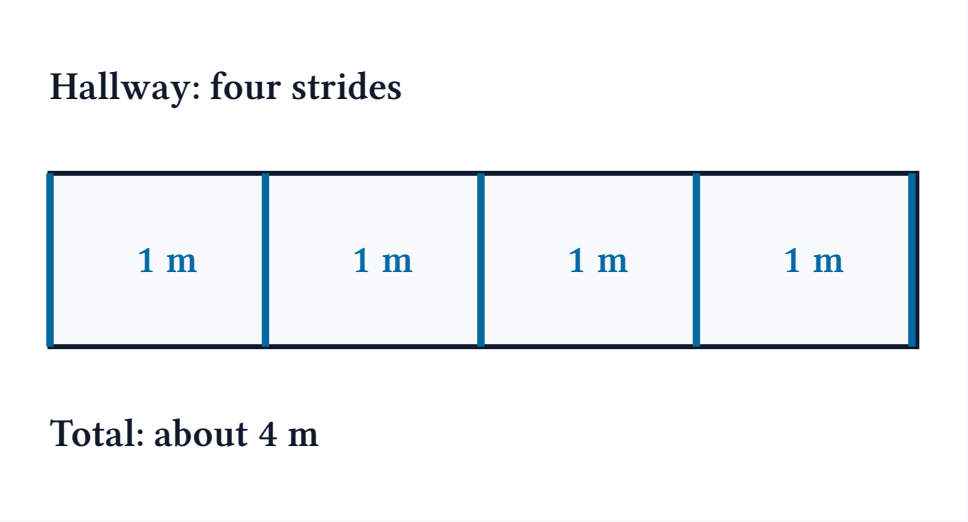
Imperial units of length

- A thumb width is about 1 inch.
- A long step is about 1 yard.
- A 20-minute walk covers about 1 mile.

Imperial length
~1 in thumb width
~1 yd long step

Four strides along a hallway

- One long stride stands for about 1 m.
- Four strides give about 4 m.



Choosing a unit

- A hallway estimate uses metres.
- The thickness of a coin uses millimetres.

Pick the unit

Hallway → metres

Coin thickness → millimetres

A window checked with a ruler

- Three fingertip widths give an estimate of 3 cm.
- The ruler reads 3.2 cm.

A ruler for a straight edge

- Place the zero mark at one end.
- Read the mark at the other end.

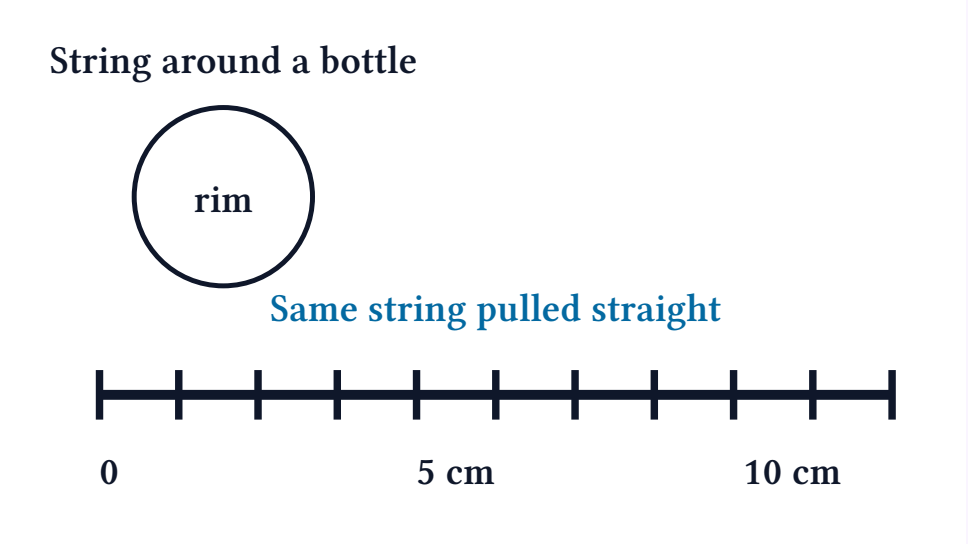
Zero at one end



Read the mark at the other end

String for a bottle rim

- Wrap string around the rim.
- Straighten that string on a ruler.



The oval base of a lamp

- Wrap string around the oval base.
- Record the tool, the path, and the unit.

Quick check

- Four shoe lengths give the estimate.
- A tape measure can check it, with zero at one end.

A length answer has three parts

- Name the referent or the tool.
- Name the count or the reading, with the unit.

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Proportional conversions between SI and imperial units

A six-foot fence

- The fence is 6 ft tall.
- The gate hardware is marked in centimetres.
- The same height needs both names.

Equal lengths, two names

- The length 1 ft is the same object as 12 in.
- The length 1 in is the same object as 2.54 cm.

Key terms

proportion

A statement that two ratios are equal, used here to change a measurement from one unit to another. ($\frac{a}{b} = \frac{c}{d}$)

conversion factor

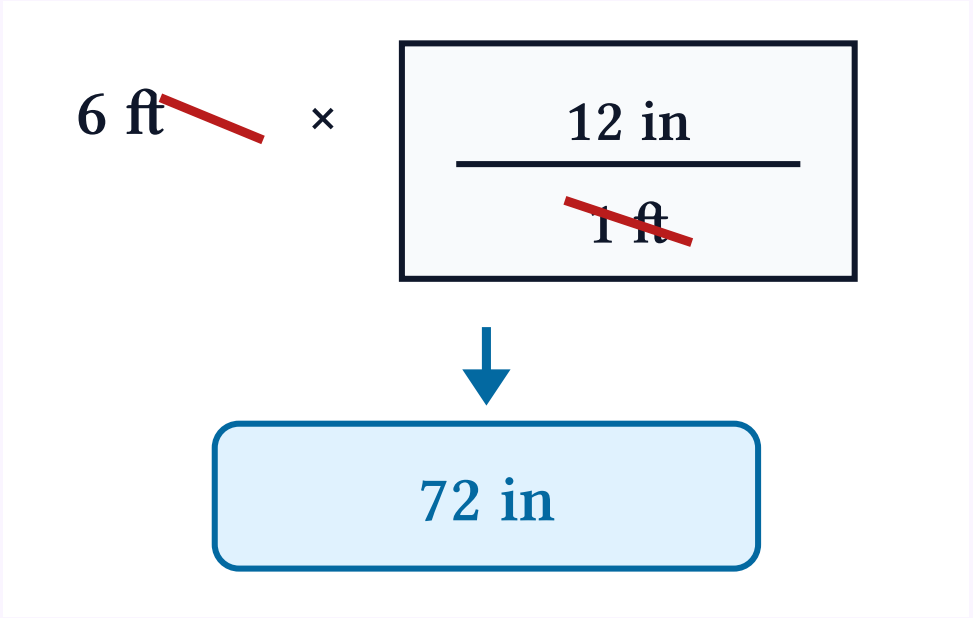
A ratio of equal lengths written in two units, arranged so the unwanted unit cancels. ($\frac{2.54\ \text{cm}}{1\ \text{in}}$)

unit analysis

A check that multiplies quantities with their units and simplifies the units that remain. (units cancel in a product)

Feet into inches

- Start with 6 ft.
- Multiply by 12 in/1 ft.
- The feet cancel and 72 in remain.



Inches into centimetres

- Start with 72 in.
- Multiply by 2.54 cm⁄1 in.
- The product is 182.88 cm.

Two steps in one line

- The fence is 6 ft.
- The chain is 12 in⁄1 ft, then 2.54 cm⁄1 in.
- The height is 182.88 cm.

6 ft fence

× 12 in/ft

× 2.54 cm/in

Miles into kilometres

- The sign says 2 mi.
- Use the rounded factor 1 mi = 1.609 km.
- The product is 3.218 km.

2 mi sign

2 × 1.609 km/mi

≈ 3.22 km

Which way the count moves

- Changing to a smaller unit raises the count.
- Changing to a larger unit lowers the count.

Count moves opposite size

Smaller unit → bigger number

Larger unit → smaller number

A 3-yard panel

- The panel is 3 yd wide.
- Use 3 ft⁄1 yd, then 12 in⁄1 ft.
- The width is 108 in.

Quick check

- Convert 5 ft to centimetres.
- Use 12 in = 1 ft and 2.54 cm = 1 in.

Conversion factors

- Write the length as a fraction over 1.
- Multiply by a factor that equals one length.

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Unit M: Measurement and trigonometry

Surface area and volume

A grain bin and its roof

- The bin is a cylinder.
- The roof is a cone on the same radius.
- Grain fills the space. Paint covers the outside.



Five solids, two measures

- Volume of a cylinder is the base area times the height.
- Volume of a matching cone is one third of that product.

Key terms

surface area

The total area of the exterior faces of a solid, measured in square units. (SA)

volume

The amount of space a solid occupies, measured in cubic units. (V)

right prism

A prism whose sides stand perpendicular to the bases, so the height meets the base at a right angle. ($V = Bh$)

right cylinder

A solid with two parallel circular bases and a curved side perpendicular to those bases. ($V = \pi r^2 h$)

Key terms

right pyramid

A solid with a polygon base and triangular faces that meet at an apex above the centre of the base. ($V = (\frac{1}{3})Bh$)

right cone

A solid with a circular base and a smooth side that meets at an apex above the centre of the base. ($V = (\frac{1}{3})\pi r^2 h$)

sphere

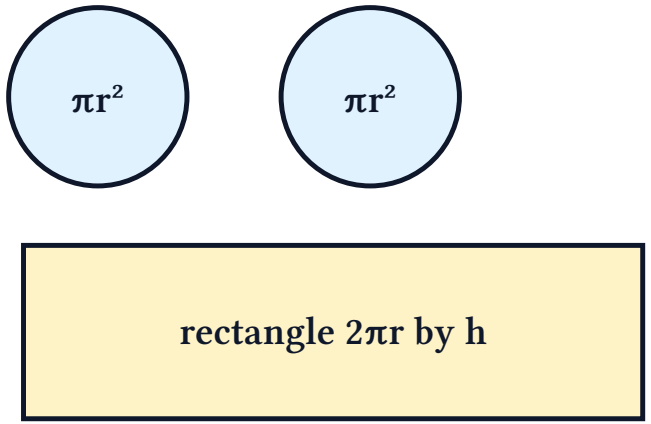
The set of points in space at a fixed distance, the radius, from a centre. ($V = (\frac{4}{3})\pi r^3$)

slant height

The distance along the sloping face from the base edge to the apex, used in cone and pyramid surface area. (s)

Cylinder bases

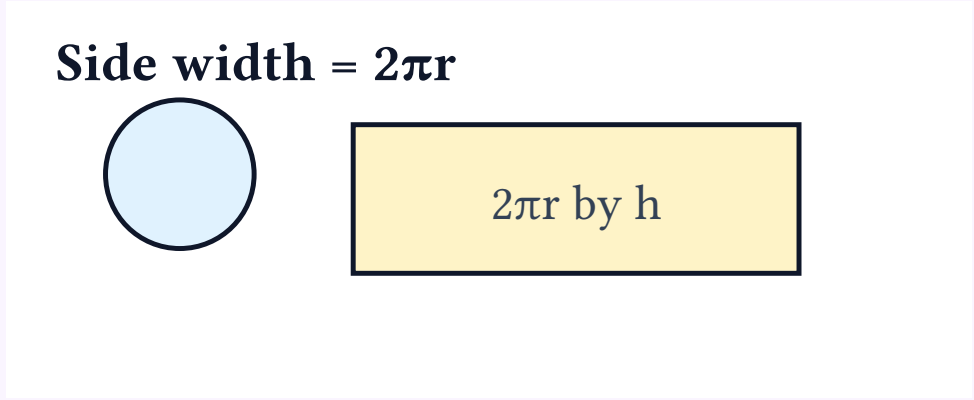
- The radius is 3 cm and $\pi = 3.14$.
- One base is $3.14 \times 9 = 28.26$ cm^2.
- Two bases are 56.52 cm^2.



Cylinder net: two bases and the side

Cylinder side

- The side is a rectangle.
- Its width is the circumference
  $2 \times 3.14 \times 3$.
- Its area is
  $18.84 \times 10 = 188.4$ cm^2.

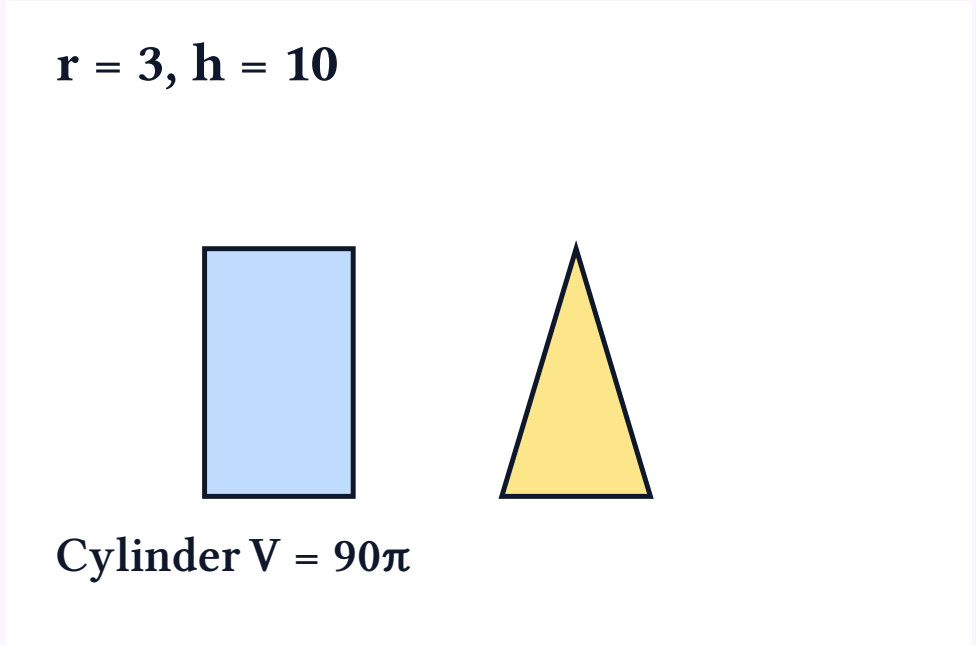


Adding the cylinder surface

- The bases contribute 56.52 cm^2.
- The side contributes 188.4 cm^2.
- The surface area is 244.92 cm^2.

Cone beside a matching cylinder

- Both solids use r = 3 and h = 10.
- The cylinder volume is 90π.
- The cone volume is 30π.



A missing cylinder height

- The volume is 100π and the radius is 5.
- The base area is 25π.
- The height is $100\pi / 25\pi = 4$.

$V = 100\pi$, $r = 5$

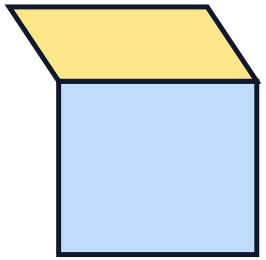
Base area $\pi r^2 = 25\pi$

$h = V \div \text{base} = 4$

Bin volume

- The cylinder part is $\pi \times 4 \times 5 = 20\pi$.
- The cone roof is one third $\times \pi \times 4 \times 3 = 4\pi$.
- The total volume is 24π.

Cylinder + cone roof



$20\pi + 4\pi = 24\pi$

Exterior paint on the bin

- The bottom is 4π.
- The cylinder side is 20π.
- The cone side is $2\pi\sqrt{13}$. The shared circle stays inside.

Quick check

- The radius is 3 cm and $\pi = 3.14$.
- The surface area is $4 \times 3.14 \times 9 = 113.04$ cm^2.
- The volume is $(\frac{4}{3}) \times 3.14 \times 27 = 113.04$ cm^3.

A formula needs matching units

- Write the known radius and height first.
- Then choose volume or surface area.

Image credits

Corey Coyle, Harvestore® Silo and Grain Bins - panoramio, CC BY 3.0

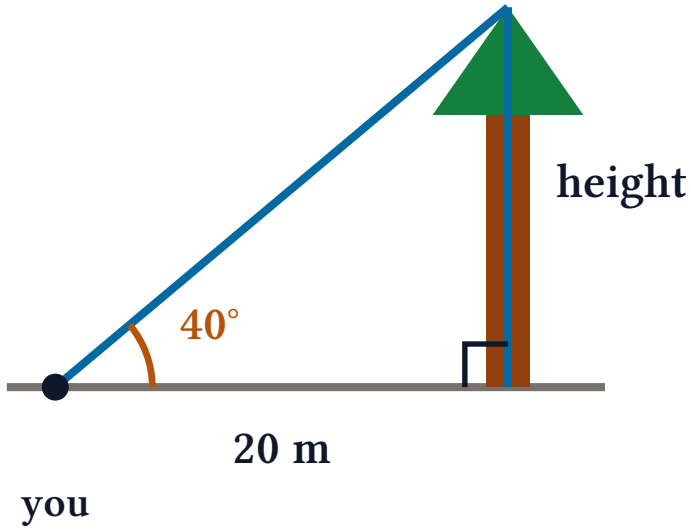
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Primary trigonometric ratios in right triangles

A tree from the ground

- You stand 20 m from the trunk.
- The angle up to the top is 40 degrees.
- The height is the opposite side.



Three ratios

- The hypotenuse sits across from the right angle.
- Sine uses opposite over hypotenuse.

Key terms

hypotenuse

The longest side of a right triangle, the side across from the right angle. (the side opposite the right angle)

opposite side

The leg across the triangle from a chosen acute angle. (across from a named acute angle)

adjacent side

The leg that touches a chosen acute angle and is the remaining side once the hypotenuse is identified. (beside a named acute angle)

sine

The ratio of the opposite side to the hypotenuse for an acute angle in a right triangle. ($\sin\theta$ = opposite/hypotenuse)

Key terms

cosine

The ratio of the adjacent side to the hypotenuse for an acute angle in a right triangle. ($\cos\theta$ = adjacent/hypotenuse)

tangent

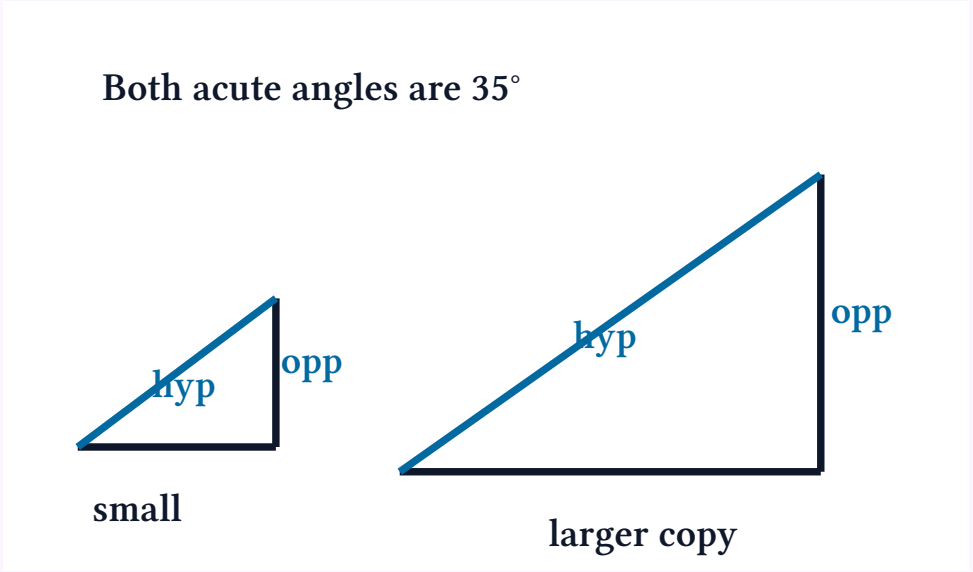
The ratio of the opposite side to the adjacent side for an acute angle in a right triangle. ($\tan\theta$ = opposite/adjacent)

angle of elevation

The angle between a horizontal line of sight and the line looking up to an object. (angle above the horizontal)

A 3-4-5 ratio that scales

- In a small triangle, opposite 3 and hypotenuse 5 give sine 0.6.
- Doubling both sides gives 6 and 10, and $^{6}/_{10}$ is still 0.6.



A 30 degree triangle

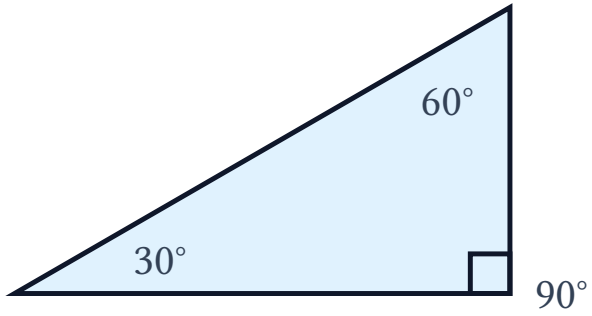
- The hypotenuse is 10 cm.
- Using $\sin 30° = 0.5$, the opposite side is 5 cm.
- Using the rounded value $\cos 30° = 0.866$, the adjacent side is 8.66 cm.

The third angle

- The right angle is 90 degrees.
- The acute pair sums to 90 degrees.
- Here the other acute angle is 60 degrees.

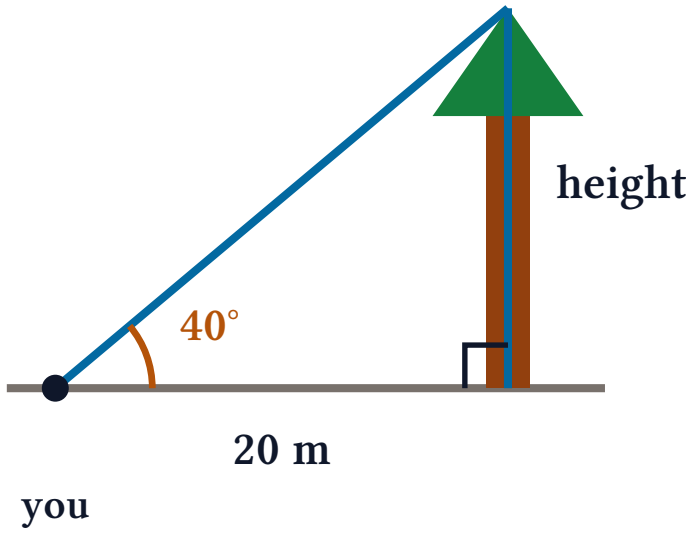
Acute angles sum to 90°

30° + 60° = 90°



Choosing tangent for the tree

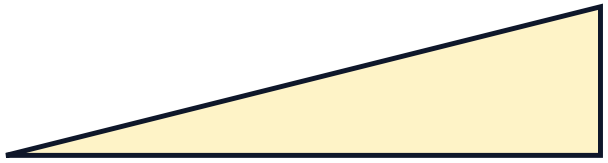
- You know the adjacent side (20 m) and want the opposite side (the height), so use tangent.
- Tangent is opposite over adjacent.
- Using the rounded value $\tan 40° = 0.839$, the height is $20 \times 0.839 = 16.78$ m.



The Pythagorean theorem for sides

- A ramp rises 0.6 m over a run of 3.6 m.
- The squares are 0.36 and 12.96.
- Their sum is 13.32, and the ramp is about 3.65 m.

Ramp 0.6 m rise, 3.6 m run



$a^2 + b^2 = c^2$

A ladder against a wall

- The ladder is 6.5 m and the base is 2.5 m out.
- The height is $\sqrt{6.5^2 - 2.5^2} = \sqrt{36} = 6m$.

Quick check

- The hypotenuse is 13 cm and one acute angle is 25 degrees.
- The opposite side uses sine: opposite = 13 × sin 25°.

A ratio or a theorem

- An angle measure points to sine, cosine, or tangent.
- Three sides and a right angle point to the Pythagorean theorem.