Algebra and number slide deck

Mathematics 10C

Unit AN: Algebra and number

Factors and roots

Tiles that finish a square

- Twelve tiles make a 3 by 4 rectangle.
- Thirty-six tiles make a 6 by 6 square.

Factors and multiples

- The prime factors of 12 are 2 and 3.
- The edge of a 6 by 6 tile square is 6.

Key terms

prime factor

A prime number that appears in a product equaling a whole number. ($84 = 2^2 \times 3 \times 7$)

greatest common factor

The largest whole number that divides each number in a set. (GCF)

least common multiple

The smallest positive whole number that is a multiple of each number in a set. (LCM)

square root

The non-negative number which multiplies by itself to give n, when n is a perfect square in this lesson. ($\sqrt{n}$)

Key terms

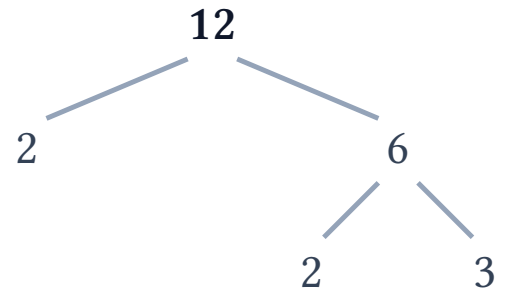
cube root

The number which multiplies by itself twice more to give n, when n is a perfect cube in this lesson. ($\sqrt[3]{n}$)

Factoring 12 and 18

- Start with $12 \div 2 = 6$, then $6 \div 2 = 3$, then $3 \div 3 = 1$.
- The prime factorization of 12 is $2^2 \times 3$.
- Start with $18 \div 2 = 9$, then $9 \div 3 = 3$, then $3 \div 3 = 1$.
- The prime factorization of 18 is 2×3^2.

Factor tree for 12



$12 = 2^2 \times 3$

GCF of 12 and 18

- The prime factorization of 12 is $12 = 2^2 \times 3$.
- The prime factorization of 18 is $18 = 2 \times 3^2$.
- The lowest shared powers are 2^1 and 3^1.
- The GCF is $2 \times 3 = 6$.

GCF from prime lists

$12 = 2^2 \times 3$

$18 = 2 \times 3^2$

$\text{GCF} = 2 \times 3 = 6$

LCM of 12 and 18

- The highest power of 2 is 2^2.
- The highest power of 3 is 3^2.
- The LCM is $2^2 \times 3^2 = 36$.

LCM from prime lists

Highest 2 power: 2^2

Highest 3 power: 3^2

LCM = $4 \times 9 = 36$

Why 0 and 1 have no prime factors

- The only positive factor of 1 is 1, and 1 sits outside the primes.
- Every product of primes is 2 or larger, so 0 has no prime factors.

0 and 1

1 has only factor 1 (not prime)

0 has every integer factor

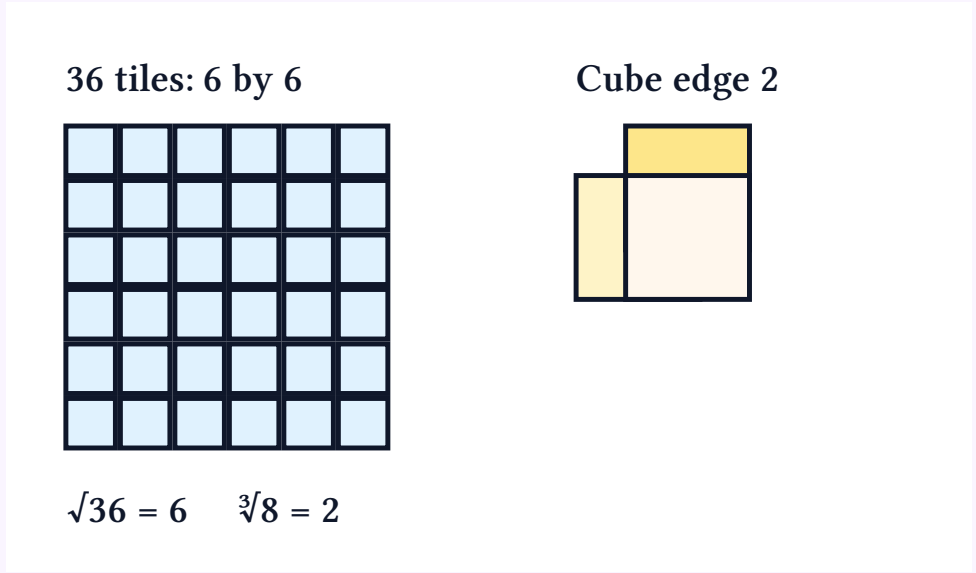
Prime factors start at 2

GCF and LCM of 84 and 90

- The factorization of 84 is $2^2 \times 3 \times 7$.
- The factorization of 90 is $2 \times 3^2 \times 5$.
- The GCF is $2 \times 3 = 6$.
- The LCM is $2^2 \times 3^2 \times 5 \times 7 = 1260$.

A square of 36 tiles

- Thirty-six tiles form a 6 by 6 square.
- The square root of 36 is 6, and $6 \times 6 = 36$.

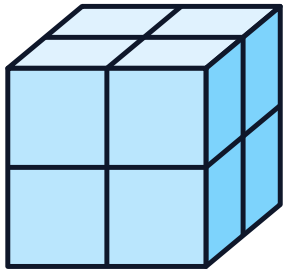


A cube of 8 and a leftover 50

- Eight unit cubes form a $2 \times 2 \times 2$ cube, so the cube root is 2.
- Fifty sits past $7^2 = 49$ and short of $8^2 = 64$.
- Fifty also sits past $3^3 = 27$ and short of $4^3 = 64$.

$2 \times 2 \times 2$ cube

8 unit cubes



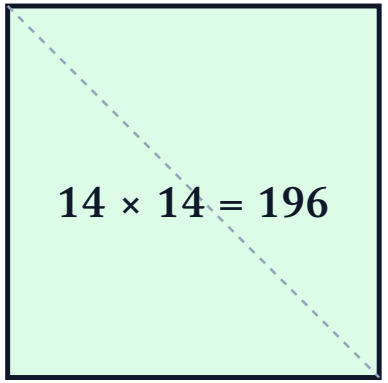
$\sqrt[3]{8} = 2$, since $2^3 = 8$

Each edge is 2 units long

A patio of 196 stones

- The product 14 × 14 = 196.
- The square root of 196 is 14.

14 by 14 patio



√196 = 14

A crate of 216 boxes

- The product $6 \times 6 \times 6 = 216$.
- The cube root of 216 is 6.

Quick check

- The GCF of 60 and 42 is $2 \times 3 = 6$.
- Twenty-seven unit cubes form a $3 \times 3 \times 3$ cube, so the cube root is 3.

Prime factorization

- The GCF of 12 and 18 is 6.
- The square root of 36 is the side 6.

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Unit AN: Algebra and number

Irrational numbers

The diagonal of a unit square

- A 1 by 1 square has diagonal $\sqrt{2}$.
- The length $\sqrt{2}$ is about 1.414, between 1 and 2.

The real number system

- The fraction ¾ is rational.
- The number $\sqrt{2}$ is irrational, and its index is 2.

Key terms

rational number

A number that can be written as a ratio of two integers with a nonzero denominator. (p/q)

irrational number

A real number whose decimal continues without a repeating block, such as $\sqrt{2}$ or π. ($\sqrt{2}$, π)

mixed radical

A radical with a coefficient outside the root sign and a radicand that has no perfect-power factor left. ($6\sqrt{2}$)

entire radical

A radical whose coefficient is 1, with every factor inside the root sign. ($\sqrt{72}$)

Key terms

index

The small number on a radical that tells how many equal factors are sought. An unwritten index means 2. (the n in $\sqrt[n]{a}$)

Ordering four numbers

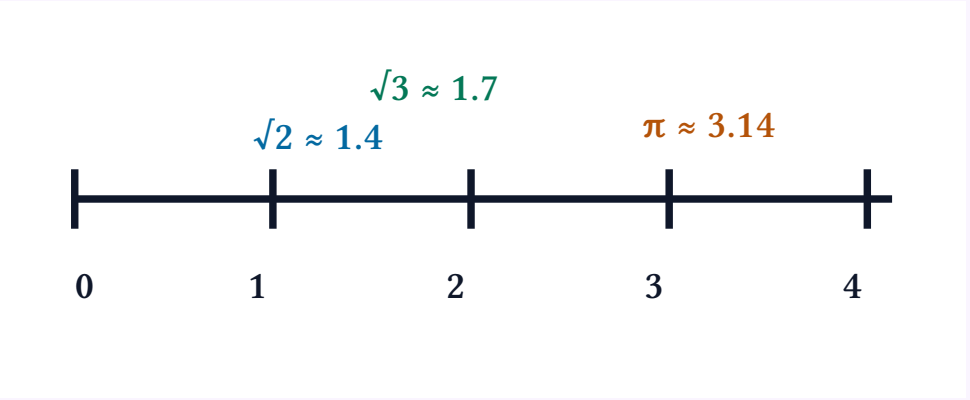
- The number 0.5 equals ½, so it is rational.
- The number $\sqrt{9}$ equals 3, so it is rational.
- The number $\sqrt{2}$ is irrational.
- The number π is irrational.

Order on a number line



Placing them on a line

- The value $\sqrt{2}$ is about 1.414.
- The value $\sqrt{3}$ is about 1.732.
- The value π is about 3.14, just past 3.



Simplifying √72

- The radical √72 equals √(36 × 2).
- The root √36 = 6, so √72 = 6√2.
- A calculator shows 6√2 is about 8.485.

The index of a radical

- The symbol $\sqrt{16}$ means the square root, index 2.
- The symbol $\sqrt[3]{8}$ means the cube root, index 3.
- The roots are $\sqrt{16} = 4$ and $\sqrt[3]{8} = 2$.

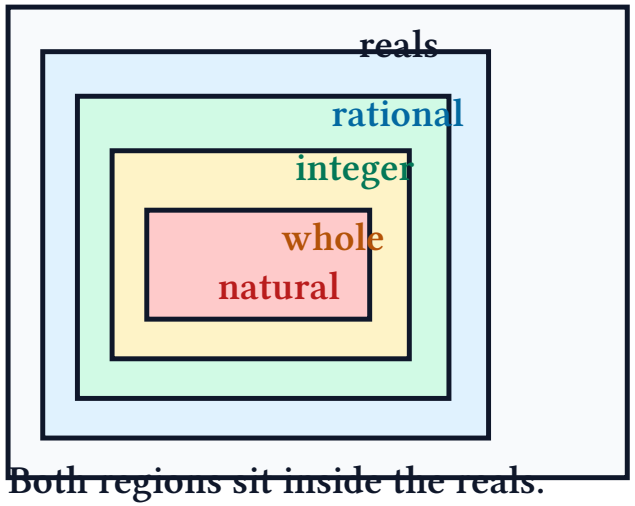
Index tells the root

$\sqrt{16}$ index 2 $\rightarrow$ 4

$\sqrt[3]{8}$ index 3 $\rightarrow$ 2

Where the sets sit

- The natural numbers in this course are 1, 2, 3, and so on.
- Whole numbers are those numbers together with 0.
- Integers add negatives such as −3. Rationals add ratios such as ¾.



Order a mixed list

- The list is ½, √2, 2, π.
- In order: ½, then √2, then 2, then π.

Quick check

- Simplify $\sqrt{50}$ as $5\sqrt{2}$.
- Place $\sqrt{2}$ to the left of 2 on the number line.

A root written two ways

- Keep $6\sqrt{2}$ as the exact simplified form.
- Use about 8.485 when you need a place on the line.

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Unit AN: Algebra and number

Integral and rational exponents

Halving a power of 2

- The value $2^3 = 8$.
- The value $2^2 = 4$.
- Each step down divides the value by 2.

Powers and roots

- In 2^3, the base is 2 and the exponent is 3.
- The rational exponent $\frac{1}{3}$ on 8 means the cube root.

Key terms

power

A product of equal factors, written with a base and an exponent. (a^n)

base

The repeated factor in a power. (the a in a^n)

exponent

The number that tells how the base is repeated, inverted, or rooted. (the n in a^n)

integral exponent

An exponent that is an integer. A negative integral exponent names a reciprocal power when the base is nonzero. ($a^{-n} = 1/a^n$)

Key terms

rational exponent

An exponent that is a ratio of integers. The unit-fraction exponent $1/n$ names the nth root when n is a positive integer. ($a^{1/n}$)

Continue the halving row

- Divide $2^1 = 2$ by 2 to get $2^0 = 1$.
- Divide $2^0 = 1$ by 2 to get $2^{-1} = \frac{1}{2}$.
- Divide $2^{-1} = \frac{1}{2}$ by 2 to get $2^{-2} = \frac{1}{4}$.
- Divide $2^{-2} = \frac{1}{4}$ by 2 to get $2^{-3} = \frac{1}{8}$.

Power	Value
2^3	8
↓ ÷ 2	
2^2	4
↓ ÷ 2	
2^1	2
↓ ÷ 2	
2^0	1
↓ ÷ 2	
2^{-1}	$\frac{1}{2}$
↓ ÷ 2	
2^{-2}	$\frac{1}{4}$
↓ ÷ 2	
2^{-3}	$\frac{1}{8}$

A base of 0

- The power $0^2 = 0$.
- The pattern for a negative exponent asks for a reciprocal.
- The reciprocal of 0 is undefined, so 0^{-2} stays undefined.

Powers of 0

$0^2 = 0$

0^{-n} needs a reciprocal (undefined here)

A unit-fraction exponent

- The value $8^{1/3}$ asks for the cube root of 8.
- The product $2 \times 2 \times 2 = 8$, so $8^{1\,/\,3} = 2$.

Multiplying powers with the same base

- The product $(2^3)(2^4)$ counts $3 + 4$ factors of 2.
- The sum of the exponents is 7.
- The value $2^7 = 128$.

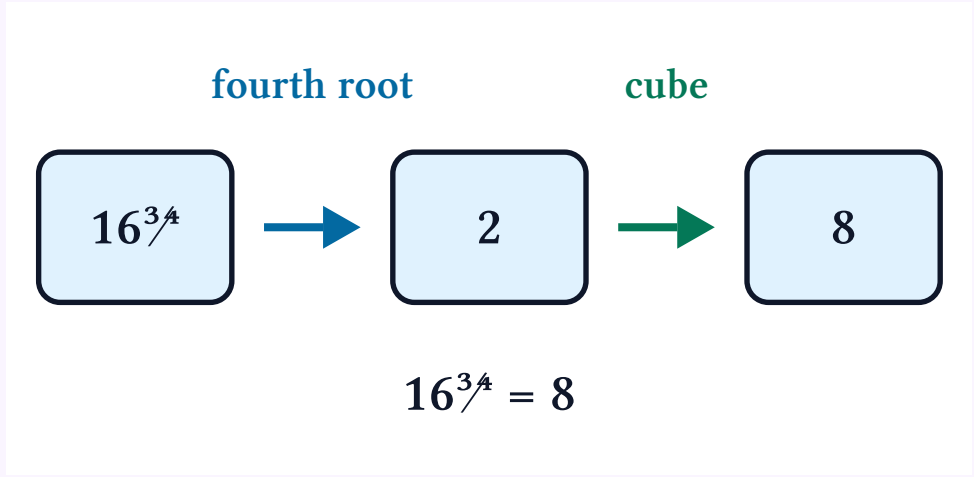
$(2^3)(2^4)$

3 + 4 factors of 2

2^7

Sixteen to the power ¾

- The exponent ¾ means a fourth root, then a cube.
- The fourth root of 16 is 2, because $2^4 = 16$.
- Then $2^3 = 8$, so $16^{3/4} = 8$.



An added-exponent slip

- The line $(x^2)^3 = x^5$ adds 2 and 3.
- The power-of-a-power law multiplies 2×3.
- The corrected power is x^6.

$(x^2)^3$

Wrong: add exponents ⟶ x^5

Correct: multiply $2 \times 3 = x^6$

A variable product

- The power $(2x^3)^2 = 4x^6$.
- Then $4x^6 \times x^{-4} = 4x^2$ when x is nonzero.

Quick check

- The value $4^{-2} = 1/16$.
- The value $27^{2/3} = 9$, because the cube root of 27 is 3 and $3^2 = 9$.

A power ready for a rectangle

- The value $2^{-3} = \frac{1}{8}$.
- The value $16^{3/4} = 8$.

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Unit AN: Algebra and number

Multiplying polynomials

A rectangle with algebra sides

- The width is $x + 3$.
- The height is $x + 2$.
- The area splits into four regions.

Terms and products

- The expression $x + 3$ has two terms.
- The product $(x + 2)(x + 3)$ multiplies every term by every term.

Key terms

monomial

A polynomial with one term, a coefficient times powers of the variables. ($3x^2$)

binomial

A polynomial with two terms. ($x + 4$)

trinomial

A polynomial with three terms. ($x^2 + 5x + 6$)

like terms

Terms whose variable parts match, so their coefficients can be added. ($5x$ and $2x$)

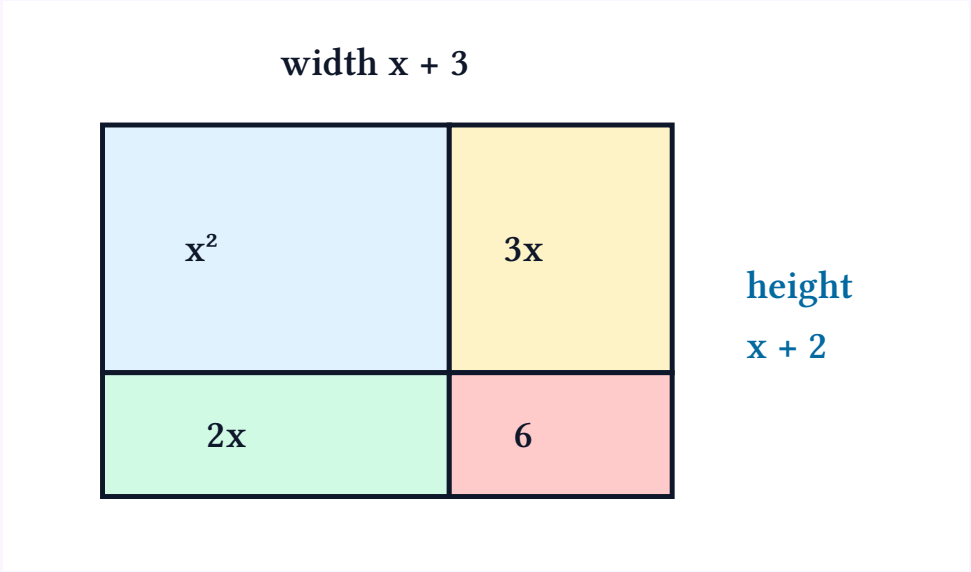
Key terms

area model

A rectangle whose side lengths are the factors and whose inner rectangles are the terms of the product. ((x + 2)(x + 3))

Four cells for (x + 2)(x + 3)

- The x by x cell is x^2.
- The x by 3 cell is 3x, and the 2 by x cell is 2x.
- The 2 by 3 cell is 6.



Combine the like terms

- The products are x^2, 3x, 2x, and 6.
- The sum 3x + 2x is 5x.
- The product is $x^2 + 5x + 6$.

Products x^2, 3x, 2x, 6

$x^2 + 5x + 6$

3x + 2x = 5x

The number cousin 23 × 14

- Write 23 as 20 + 3 and 14 as 10 + 4.
- The four products are 200, 80, 30, and 12.
- The sum is 322.

Every pair in $(2x + 1)$(x + 4)

- The four products are $2x^2$, 8x, x, and 4.
- The like terms 8x and x make 9x.
- The product is $2x^2 + 9x + 4$.

(2x+1)(x+4)

$2x^2$, 8x, x, 4

$2x^2 + 9x + 4$

Checking by substituting 3

- The factors at x = 3 are 7 and 7, and 7 × 7 = 49.
- The expanded form 2(9) + 9(3) + 4 = 18 + 27 + 4 = 49.

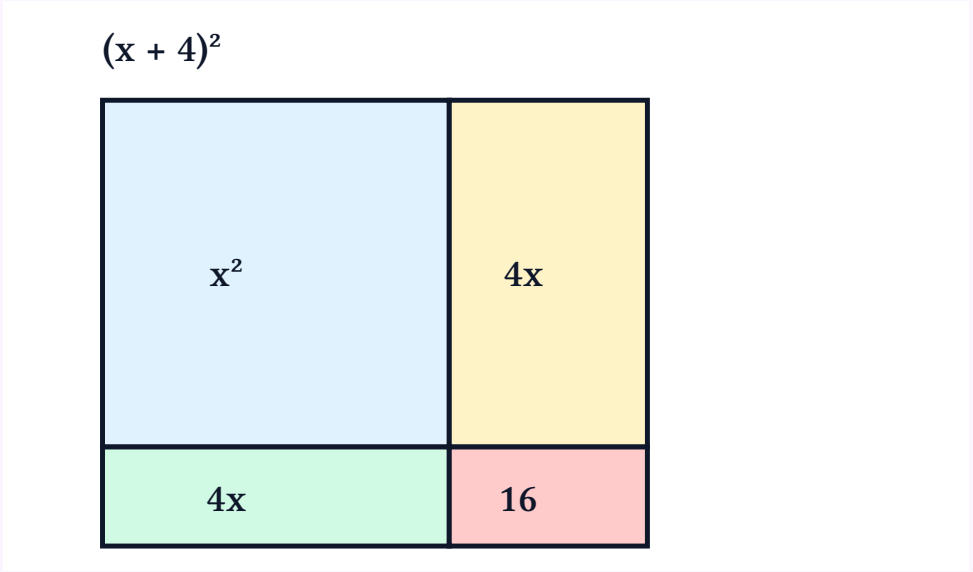
Substitute x = 3

(7)(7) = 49

Expanded form also 49

The square $(x + 4)^2$

- The four products are x^2, 4x, 4x, and 16.
- The like terms make 8x.
- The product is $x^2 + 8x + 16$.



A longer product

- Multiply $(x + 1)$ by x^2, by $2x$, and by 3.
- The products are x^3, $2x^2$, $3x$, x^2, $2x$, and 3.
- The sum is $x^3 + 3x^2 + 5x + 3$.

Quick check

- The product $(3x + 2)(x + 5)$ is $3x^2 + 17x + 10$.
- The slip $x^2 + 16$ for $(x + 4)^2$ drops the cross terms $4x$ and $4x$.

Checking a product

- The product $(x + 2)(x + 3)$ is $x^2 + 5x + 6$.
- Factoring will look for the sides that rebuild that trinomial.

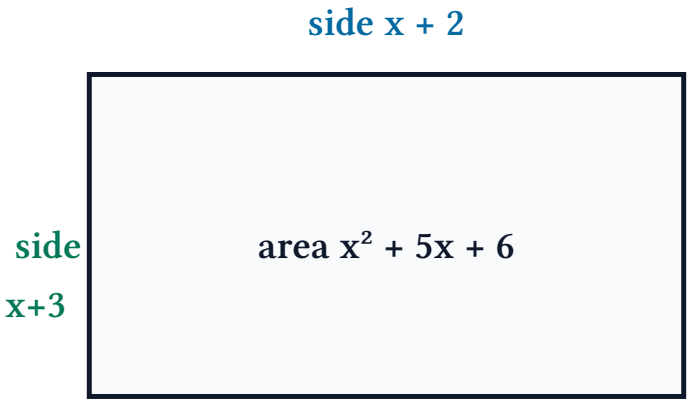
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Unit AN: Algebra and number

Factoring polynomials

Factoring trinomials

- The area $x^2 + 5x + 6$ came from sides $x + 2$ and $x + 3$.
- Factoring hunts for those sides.



Factors read from the sides: $(x + 2)(x + 3)$

Shared factors and rectangles

- The binomial $6x^2 + 9x$ has a common factor $3x$.
- The trinomial $x^2 + 5x + 6$ factors as a rectangle of binomial sides.

Key terms

common factor

A factor shared by every term of a polynomial, written outside the brackets. (3x)

factored form

A polynomial written as a product of polynomials. (3x(2x + 3))

difference of squares

A binomial whose terms are squares separated by subtraction, the trinomial case with middle coefficient 0. (x^2 - 16)

trinomial

A polynomial with three terms, factored here by a pair of numbers. (x^2 + 5x + 6)

A common factor first

- The terms $6x^2$ and $9x$ share $3x$.
- The factored form is $3x(2x + 3)$.

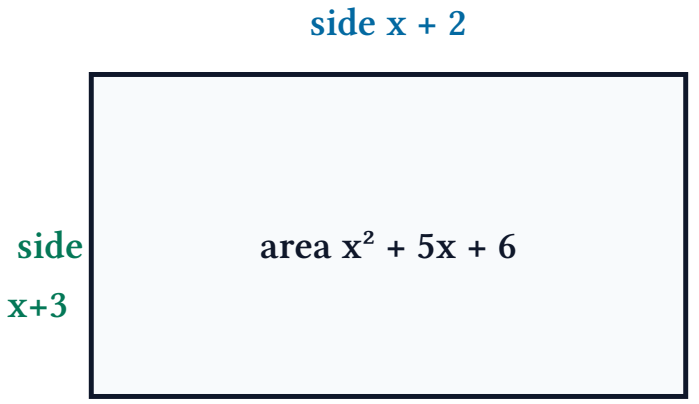
Factor $6x^2 + 9x$

GCF: $3x$

$3x(2x + 3)$

Two numbers for $x^2 + 5x + 6$

- The pair needs sum 5 and product 6.
- The numbers 2 and 3 work, because $2 + 3 = 5$ and $2 \times 3 = 6$.
- The factors are $(x + 2)(x + 3)$.



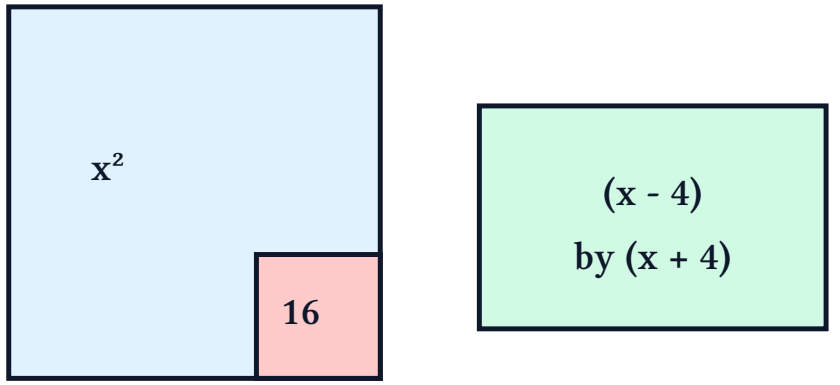
Factors read from the sides: $(x + 2)(x + 3)$

Checking by multiplication

- The product $(x + 2)(x + 3)$ is $x^2 + 3x + 2x + 6$.
- The like terms make $x^2 + 5x + 6$.

A difference of squares

- The polynomial $x^2 - 16$ is $x^2 + 0x - 16$.
- The pair -4 and 4 has sum 0 and product -16.
- The factors are $(x - 4)(x + 4)$.



$x^2 - 16$ equals that rectangle

A pair that fails

- The claim uses $(x + 1)(x + 6)$ for $x^2 + 5x + 6$.
- The sum $1 + 6$ is 7, and the product is 6.
- The expansion is $x^2 + 7x + 6$, a different middle term.

Check $(x + 1)(x + 6)$

Sum: $1 + 6 = 7$

Need sum 5 for $x^2 + 5x + 6$

Use factors with product 6 and sum 5

$2 \times 3 = 6$, $2 + 3 = 5$: $(x + 2)(x + 3)$

Factoring $x^2 - 9$

- The middle coefficient is 0.
- The pair -3 and 3 has product -9.
- The factors are $(x - 3)(x + 3)$.

Quick check

- The polynomial $4x^2 - 10x$ factors as $2x(2x - 5)$.
- The polynomial $x^2 - 16$ factors as $(x - 4)(x + 4)$.

Checking a factorization

- The factors of $x^2 + 5x + 6$ are $(x + 2)(x + 3)$.
- Multiplying those sides returns the trinomial.