Mathematics 10C

Unit AN: Algebra and number

Irrational numbers

The diagonal of a unit square

- A 1 by 1 square has diagonal $\sqrt{2}$.
- The length $\sqrt{2}$ is about 1.414, between 1 and 2.

The real number system

- The fraction ¾ is rational.
- The number $\sqrt{2}$ is irrational, and its index is 2.

Key terms

rational number

A number that can be written as a ratio of two integers with a nonzero denominator. (p/q)

irrational number

A real number whose decimal continues without a repeating block, such as $\sqrt{2}$ or π. ($\sqrt{2}$, π)

mixed radical

A radical with a coefficient outside the root sign and a radicand that has no perfect-power factor left. ($6\sqrt{2}$)

entire radical

A radical whose coefficient is 1, with every factor inside the root sign. ($\sqrt{72}$)

Key terms

index

The small number on a radical that tells how many equal factors are sought. An unwritten index means 2. (the n in $\sqrt[n]{a}$)

Ordering four numbers

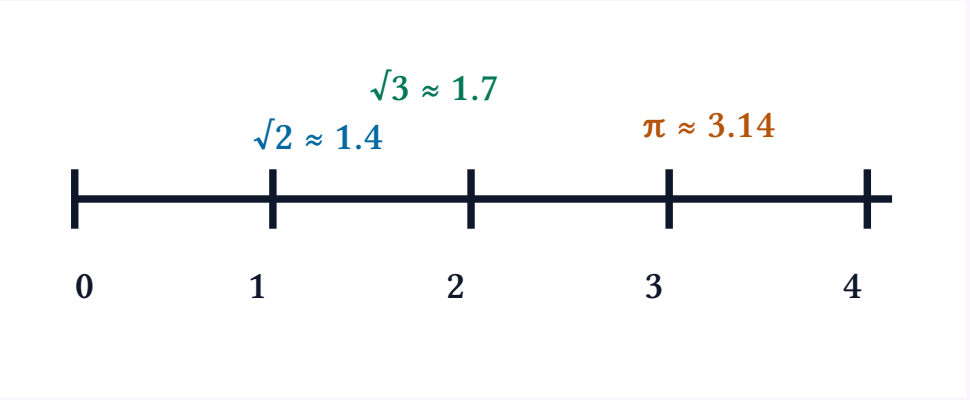
- The number 0.5 equals ½, so it is rational.
- The number $\sqrt{9}$ equals 3, so it is rational.
- The number $\sqrt{2}$ is irrational.
- The number π is irrational.

Order on a number line



Placing them on a line

- The value $\sqrt{2}$ is about 1.414.
- The value $\sqrt{3}$ is about 1.732.
- The value π is about 3.14, just past 3.



Simplifying √72

- The radical √72 equals √(36 × 2).
- The root √36 = 6, so √72 = 6√2.
- A calculator shows 6√2 is about 8.485.

The index of a radical

- The symbol $\sqrt{16}$ means the square root, index 2.
- The symbol $\sqrt[3]{8}$ means the cube root, index 3.
- The roots are $\sqrt{16} = 4$ and $\sqrt[3]{8} = 2$.

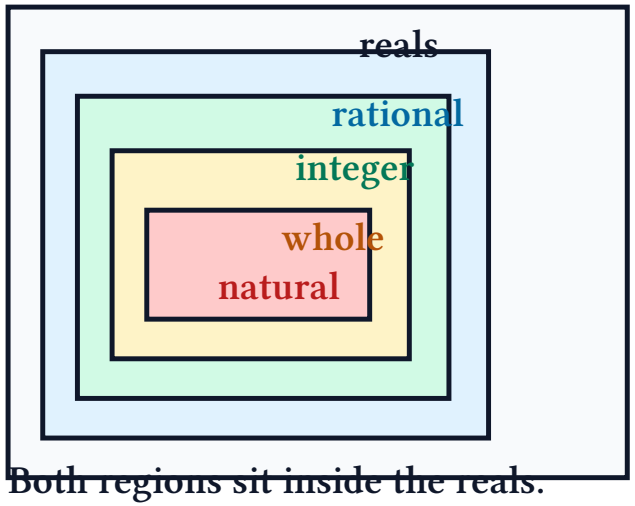
Index tells the root

$\sqrt{16}$ index 2 $\rightarrow$ 4

$\sqrt[3]{8}$ index 3 $\rightarrow$ 2

Where the sets sit

- The natural numbers in this course are 1, 2, 3, and so on.
- Whole numbers are those numbers together with 0.
- Integers add negatives such as −3. Rationals add ratios such as ¾.



Order a mixed list

- The list is ½, √2, 2, π.
- In order: ½, then √2, then 2, then π.

Quick check

- Simplify $\sqrt{50}$ as $5\sqrt{2}$.
- Place $\sqrt{2}$ to the left of 2 on the number line.

A root written two ways

- Keep $6\sqrt{2}$ as the exact simplified form.
- Use about 8.485 when you need a place on the line.