Mathematics 10C

Unit AN: Algebra and number

Integral and rational exponents

Halving a power of 2

- The value $2^3 = 8$.
- The value $2^2 = 4$.
- Each step down divides the value by 2.

Powers and roots

- In 2^3, the base is 2 and the exponent is 3.
- The rational exponent $\frac{1}{3}$ on 8 means the cube root.

Key terms

power

A product of equal factors, written with a base and an exponent. (a^n)

base

The repeated factor in a power. (the a in a^n)

exponent

The number that tells how the base is repeated, inverted, or rooted. (the n in a^n)

integral exponent

An exponent that is an integer. A negative integral exponent names a reciprocal power when the base is nonzero. ($a^{-n} = 1/a^n$)

Key terms

rational exponent

An exponent that is a ratio of integers. The unit-fraction exponent $1/n$ names the nth root when n is a positive integer. ($a^{1/n}$)

Continue the halving row

- Divide $2^1 = 2$ by 2 to get $2^0 = 1$.
- Divide $2^0 = 1$ by 2 to get $2^{-1} = \frac{1}{2}$.
- Divide $2^{-1} = \frac{1}{2}$ by 2 to get $2^{-2} = \frac{1}{4}$.
- Divide $2^{-2} = \frac{1}{4}$ by 2 to get $2^{-3} = \frac{1}{8}$.

Power	Value
2^3	8
↓ ÷ 2	
2^2	4
↓ ÷ 2	
2^1	2
↓ ÷ 2	
2^0	1
↓ ÷ 2	
2^{-1}	$\frac{1}{2}$
↓ ÷ 2	
2^{-2}	$\frac{1}{4}$
↓ ÷ 2	
2^{-3}	$\frac{1}{8}$

A base of 0

- The power $0^2 = 0$.
- The pattern for a negative exponent asks for a reciprocal.
- The reciprocal of 0 is undefined, so 0^{-2} stays undefined.

Powers of 0

$0^2 = 0$

0^{-n} needs a reciprocal (undefined here)

A unit-fraction exponent

- The value $8^{1/3}$ asks for the cube root of 8.
- The product $2 \times 2 \times 2 = 8$, so $8^{1\,/\,3} = 2$.

Multiplying powers with the same base

- The product $(2^3)(2^4)$ counts 3 + 4 factors of 2.
- The sum of the exponents is 7.
- The value $2^7 = 128$.

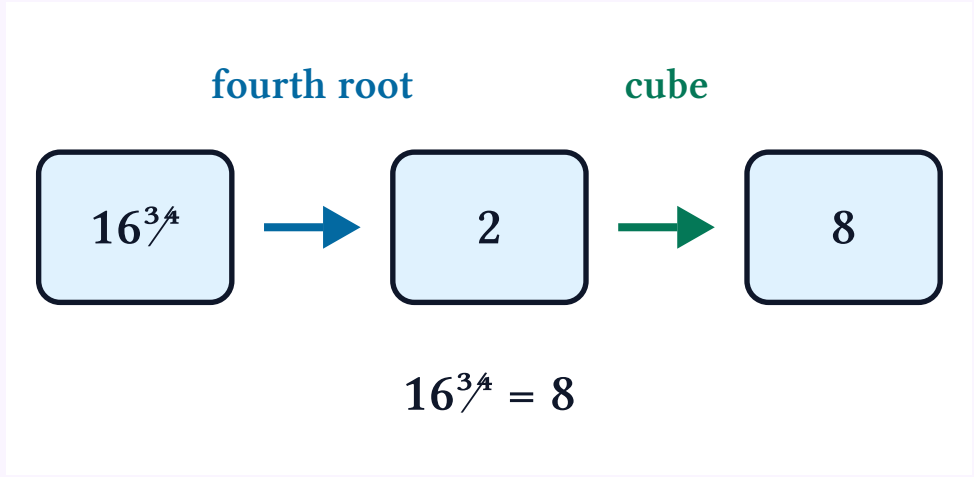
$(2^3)(2^4)$

3 + 4 factors of 2

2^7

Sixteen to the power ¾

- The exponent ¾ means a fourth root, then a cube.
- The fourth root of 16 is 2, because $2^4 = 16$.
- Then $2^3 = 8$, so $16^{3/4} = 8$.



An added-exponent slip

- The line $(x^2)^3 = x^5$ adds 2 and 3.
- The power-of-a-power law multiplies 2×3.
- The corrected power is x^6.

$(x^2)^3$

Wrong: add exponents ⟶ x^5

Correct: multiply $2 \times 3 = x^6$

A variable product

- The power $(2x^3)^2 = 4x^6$.
- Then $4x^6 \times x^{-4} = 4x^2$ when x is nonzero.

Quick check

- The value $4^{-2} = 1/16$.
- The value $27^{2/3} = 9$, because the cube root of 27 is 3 and $3^2 = 9$.

A power ready for a rectangle

- The value $2^{-3} = \frac{1}{8}$.
- The value $16^{3/4} = 8$.