# Mathematics 10C full slide deck

Mathematics 10C

## Unit M: Measurement and trigonometry

# Linear measurement with referents and tools

# A doorway and a thumb

- A doorway is about 1 m across.
- A thumb is about 1 inch across.
- A couch wider than that doorway needs another path.

# Unit referents

- A long stride stands for about 1 m.
- A size-ten shoe stands for about 1 foot.

# Key terms

**referent**

A familiar object or body length that stands in for a unit before a measuring tool is used. (a familiar length)

**SI unit**

A length unit in the International System, built by powers of ten from the metre. (mm, cm, m, km)

**imperial unit**

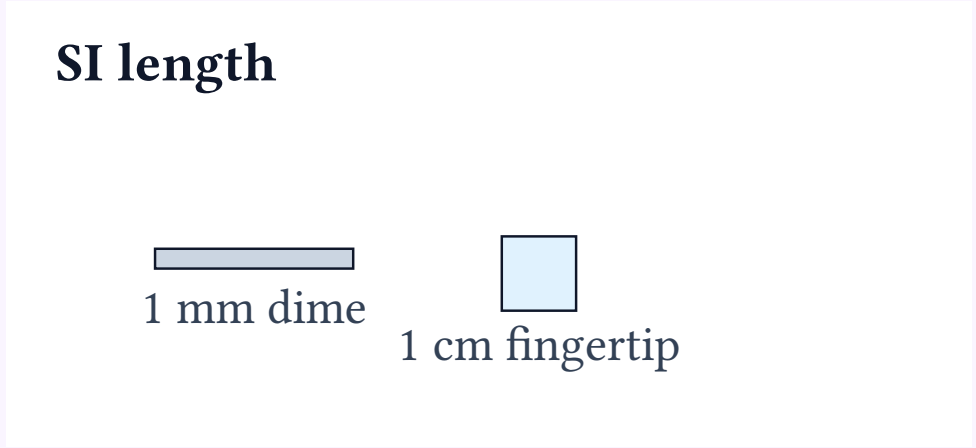
A length unit in the inch-foot-yard-mile system still used on many North American tools and signs. (in, ft, yd, mi)

**linear measurement**

The size of a distance along a line, a curve, or the edge of an object, reported with a unit. (a one-dimensional length)

# SI units of length

- A dime is about 1 mm thick.
- A fingertip is about 1 cm across.
- A brisk 12-minute walk covers about 1 km.



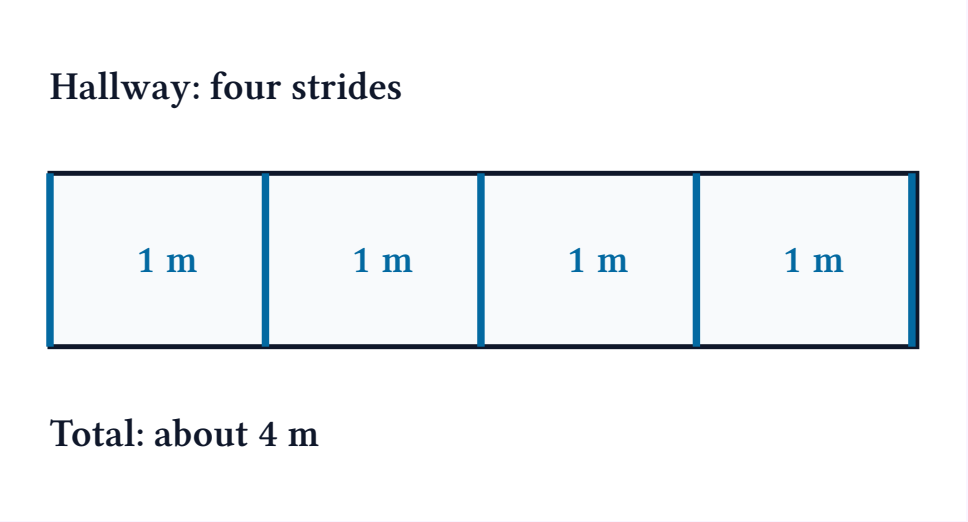
# Imperial units of length

- A thumb width is about 1 inch.
- A long step is about 1 yard.
- A 20-minute walk covers about 1 mile.

| Imperial length |
| --- |
| ~1 in thumb width |
| ~1 yd long step |

# Four strides along a hallway

- One long stride stands for about 1 m.
- Four strides give about 4 m.



# Choosing a unit

- A hallway estimate uses metres.
- The thickness of a coin uses millimetres.

**Pick the unit**

**Hallway → metres**

**Coin thickness → millimetres**

# A window checked with a ruler

- Three fingertip widths give an estimate of 3 cm.
- The ruler reads 3.2 cm.

# A ruler for a straight edge

- Place the zero mark at one end.
- Read the mark at the other end.

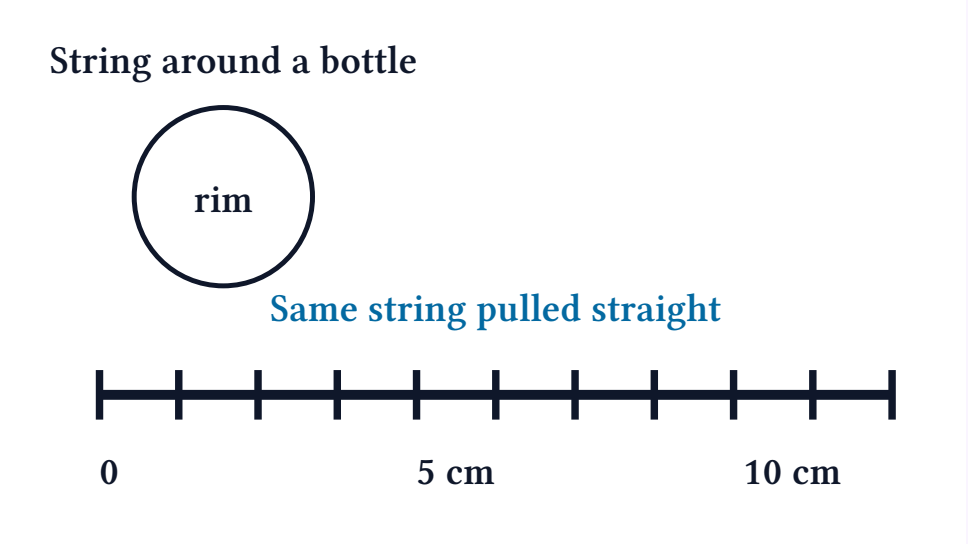
**Zero at one end**



Read the mark at the other end

# String for a bottle rim

- Wrap string around the rim.
- Straighten that string on a ruler.



# The oval base of a lamp

- Wrap string around the oval base.
- Record the tool, the path, and the unit.

## Quick check

- Four shoe lengths give the estimate.
- A tape measure can check it, with zero at one end.

# A length answer has three parts

- Name the referent or the tool.
- Name the count or the reading, with the unit.

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## Unit M: Measurement and trigonometry

# Proportional conversions between SI and imperial units

# A six-foot fence

- The fence is 6 ft tall.
- The gate hardware is marked in centimetres.
- The same height needs both names.

# Equal lengths, two names

- The length 1 ft is the same object as 12 in.
- The length 1 in is the same object as 2.54 cm.

# Key terms

**proportion**

A statement that two ratios are equal, used here to change a measurement from one unit to another. ($\frac{a}{b} = \frac{c}{d}$)

**conversion factor**

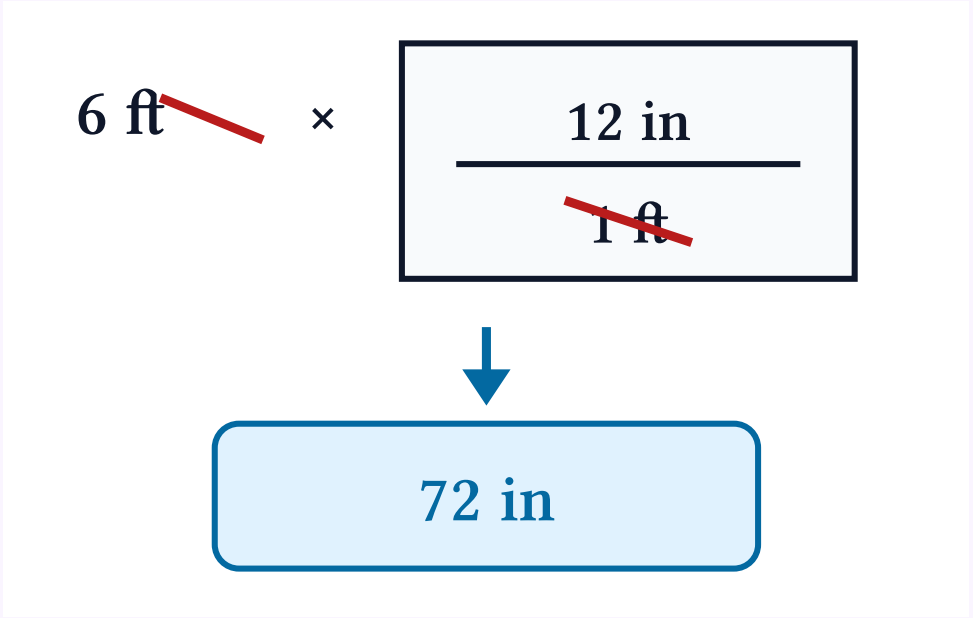
A ratio of equal lengths written in two units, arranged so the unwanted unit cancels. ($\frac{2.54\ \text{cm}}{1\ \text{in}}$)

**unit analysis**

A check that multiplies quantities with their units and simplifies the units that remain. (units cancel in a product)

# Feet into inches

- Start with 6 ft.
- Multiply by 12 in/1 ft.
- The feet cancel and 72 in remain.



# Inches into centimetres

- Start with 72 in.
- Multiply by 2.54 cm⁄1 in.
- The product is 182.88 cm.

# Two steps in one line

- The fence is 6 ft.
- The chain is 12 in⁄1 ft, then 2.54 cm⁄1 in.
- The height is 182.88 cm.

**6 ft fence**

**× 12 in/ft**

**× 2.54 cm/in**

# Miles into kilometres

- The sign says 2 mi.
- Use the rounded factor 1 mi = 1.609 km.
- The product is 3.218 km.

**2 mi sign**

**2 × 1.609 km/mi**

**≈ 3.22 km**

# Which way the count moves

- Changing to a smaller unit raises the count.
- Changing to a larger unit lowers the count.

**Count moves opposite size**

**Smaller unit → bigger number**

**Larger unit → smaller number**

## A 3-yard panel

- The panel is 3 yd wide.
- Use 3 ft⁄1 yd, then 12 in⁄1 ft.
- The width is 108 in.

## Quick check

- Convert 5 ft to centimetres.
- Use 12 in = 1 ft and 2.54 cm = 1 in.

# Conversion factors

- Write the length as a fraction over 1.
- Multiply by a factor that equals one length.

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## Unit M: Measurement and trigonometry

# Surface area and volume

# A grain bin and its roof

- The bin is a cylinder.
- The roof is a cone on the same radius.
- Grain fills the space. Paint covers the outside.



## Five solids, two measures

- Volume of a cylinder is the base area times the height.
- Volume of a matching cone is one third of that product.

# Key terms

**surface area**

The total area of the exterior faces of a solid, measured in square units. (SA)

**volume**

The amount of space a solid occupies, measured in cubic units. (V)

**right prism**

A prism whose sides stand perpendicular to the bases, so the height meets the base at a right angle. ($V = Bh$)

**right cylinder**

A solid with two parallel circular bases and a curved side perpendicular to those bases. ($V = \pi r^2 h$)

# Key terms

### right pyramid

A solid with a polygon base and triangular faces that meet at an apex above the centre of the base. ($V = (\frac{1}{3})Bh$)

### right cone

A solid with a circular base and a smooth side that meets at an apex above the centre of the base. ($V = (\frac{1}{3})\pi r^2 h$)

### sphere

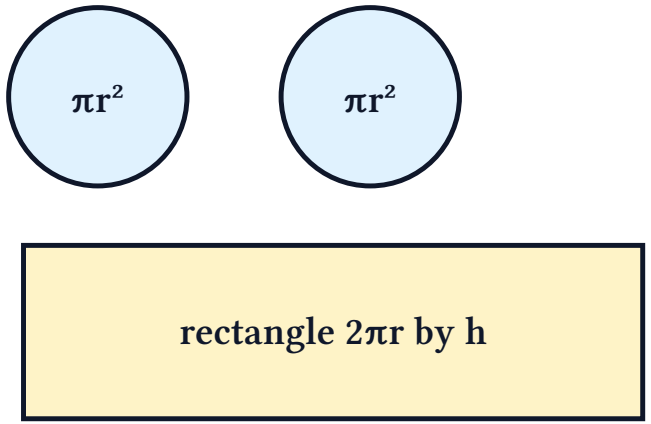
The set of points in space at a fixed distance, the radius, from a centre. ($V = (\frac{4}{3})\pi r^3$)

### slant height

The distance along the sloping face from the base edge to the apex, used in cone and pyramid surface area. ($s$)

# Cylinder bases

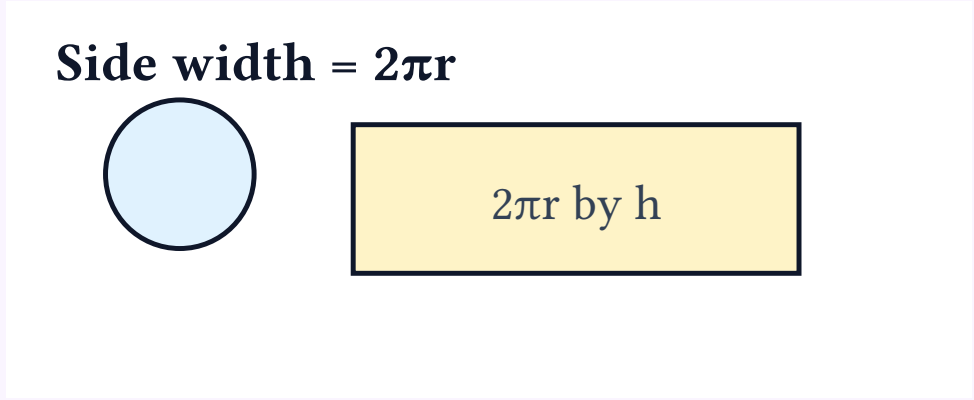
- The radius is 3 cm and $\pi = 3.14$.
- One base is $3.14 \times 9 = 28.26$ cm$^2$.
- Two bases are 56.52 cm$^2$.




Cylinder net: two bases and the side

# Cylinder side

- The side is a rectangle.
- Its width is the circumference
  $2 \times 3.14 \times 3$.
- Its area is
  $18.84 \times 10 = 188.4$ cm$^2$.

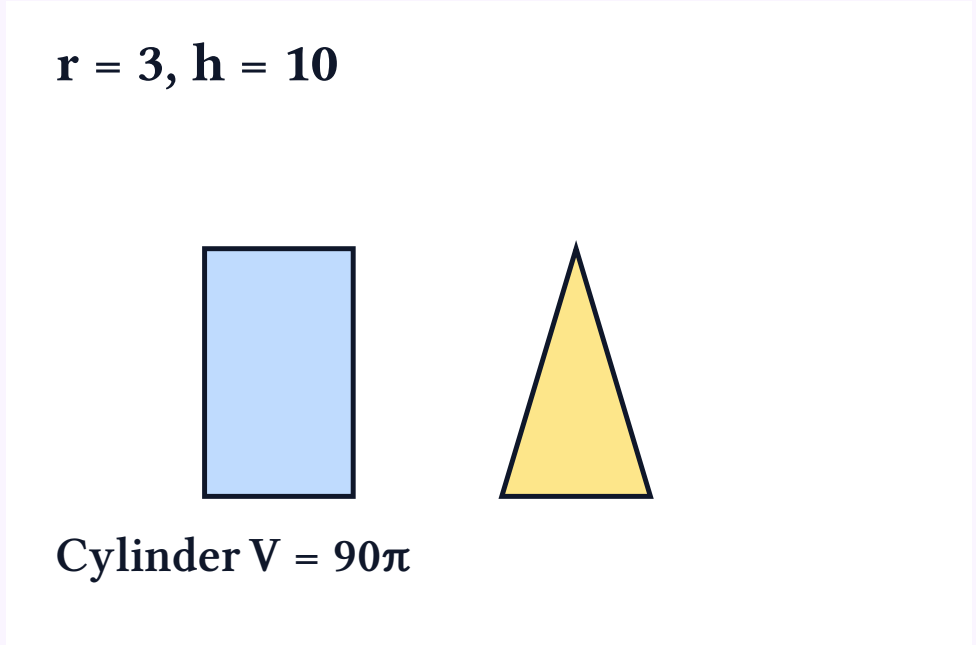


# Adding the cylinder surface

- The bases contribute 56.52 $cm^2$.
- The side contributes 188.4 $cm^2$.
- The surface area is 244.92 $cm^2$.

# Cone beside a matching cylinder

- Both solids use r = 3 and h = 10.
- The cylinder volume is 90π.
- The cone volume is 30π.



# A missing cylinder height

- The volume is $100\pi$ and the radius is 5.
- The base area is $25\pi$.
- The height is $100\pi / 25\pi = 4$.

**$V = 100\pi$, $r = 5$**

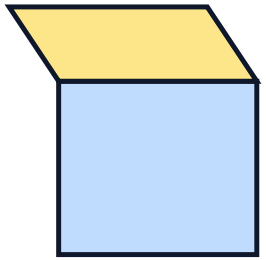
**Base area $\pi r^2 = 25\pi$**

**$h = V \div \text{base} = 4$**

# Bin volume

- The cylinder part is $\pi \times 4 \times 5 = 20\pi$.
- The cone roof is one third $\times \pi \times 4 \times 3 = 4\pi$.
- The total volume is $24\pi$.

**Cylinder + cone roof**



$20\pi + 4\pi = 24\pi$

## Exterior paint on the bin

- The bottom is $4\pi$.
- The cylinder side is $20\pi$.
- The cone side is $2\pi\sqrt{13}$. The shared circle stays inside.

## Quick check

- The radius is 3 cm and $\pi = 3.14$.
- The surface area is $4 \times 3.14 \times 9 = 113.04$ cm$^2$.
- The volume is $(\frac{4}{3}) \times 3.14 \times 27 = 113.04$ cm$^3$.

# A formula needs matching units

- Write the known radius and height first.
- Then choose volume or surface area.

# Image credits

Corey Coyle, Harvestore® Silo and Grain Bins - panoramio, CC BY 3.0

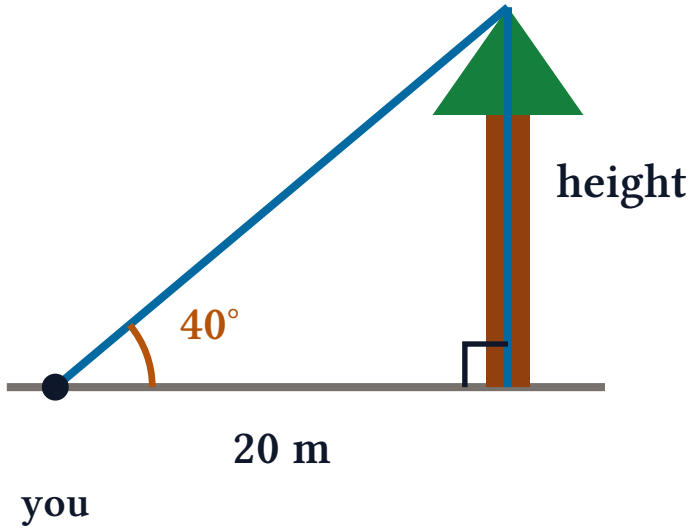
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## Unit M: Measurement and trigonometry

# Primary trigonometric ratios in right triangles

# A tree from the ground

- You stand 20 m from the trunk.
- The angle up to the top is 40 degrees.
- The height is the opposite side.



## Three ratios

- The hypotenuse sits across from the right angle.
- Sine uses opposite over hypotenuse.

# Key terms

### hypotenuse

The longest side of a right triangle, the side across from the right angle. (the side opposite the right angle)

### opposite side

The leg across the triangle from a chosen acute angle. (across from a named acute angle)

### adjacent side

The leg that touches a chosen acute angle and is the remaining side once the hypotenuse is identified. (beside a named acute angle)

### sine

The ratio of the opposite side to the hypotenuse for an acute angle in a right triangle. ($\sin\theta$ = opposite/hypotenuse)

# Key terms

**cosine**

The ratio of the adjacent side to the hypotenuse for an acute angle in a right triangle. ($\cos\theta$ = adjacent/hypotenuse)

**tangent**

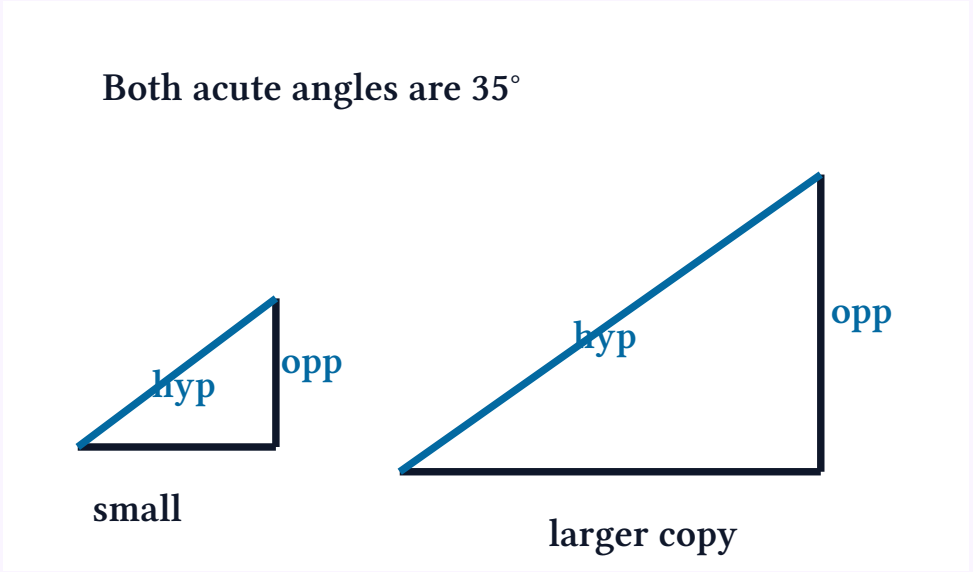
The ratio of the opposite side to the adjacent side for an acute angle in a right triangle. ($\tan\theta$ = opposite/adjacent)

**angle of elevation**

The angle between a horizontal line of sight and the line looking up to an object. (angle above the horizontal)

# A 3-4-5 ratio that scales

- In a small triangle, opposite 3 and hypotenuse 5 give sine 0.6.
- Doubling both sides gives 6 and 10, and $^{6}/_{10}$ is still 0.6.



# A 30 degree triangle

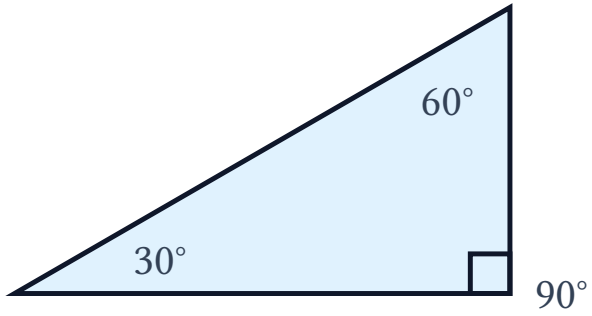
- The hypotenuse is 10 cm.
- Using $\sin 30° = 0.5$, the opposite side is 5 cm.
- Using the rounded value $\cos 30° = 0.866$, the adjacent side is 8.66 cm.

# The third angle

- The right angle is 90 degrees.
- The acute pair sums to 90 degrees.
- Here the other acute angle is 60 degrees.

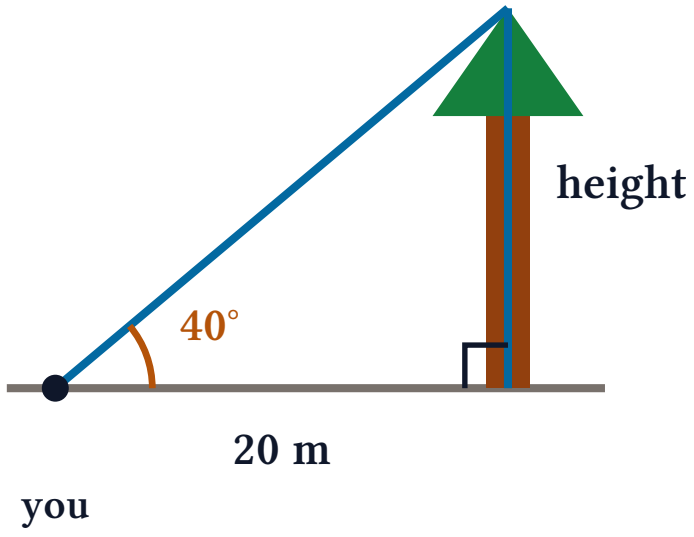
**Acute angles sum to 90°**

30° + 60° = 90°



# Choosing tangent for the tree

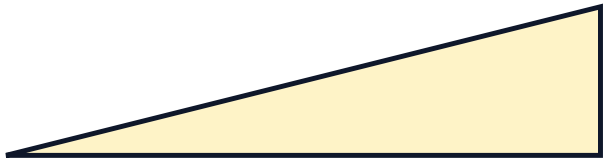
- You know the adjacent side (20 m) and want the opposite side (the height), so use tangent.
- Tangent is opposite over adjacent.
- Using the rounded value $\tan 40° = 0.839$, the height is $20 \times 0.839 = 16.78$ m.



# The Pythagorean theorem for sides

- A ramp rises 0.6 m over a run of 3.6 m.
- The squares are 0.36 and 12.96.
- Their sum is 13.32, and the ramp is about 3.65 m.

**Ramp 0.6 m rise, 3.6 m run**



$a^2 + b^2 = c^2$

# A ladder against a wall

- The ladder is 6.5 m and the base is 2.5 m out.
- The height is $\sqrt{6.5^2 - 2.5^2} = \sqrt{36} = 6m$.

## Quick check

- The hypotenuse is 13 cm and one acute angle is 25 degrees.
- The opposite side uses sine: opposite = 13 × sin 25°.

# A ratio or a theorem

- An angle measure points to sine, cosine, or tangent.
- Three sides and a right angle point to the Pythagorean theorem.

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## Unit AN: Algebra and number

# Factors and roots

# Tiles that finish a square

- Twelve tiles make a 3 by 4 rectangle.
- Thirty-six tiles make a 6 by 6 square.

# Factors and multiples

- The prime factors of 12 are 2 and 3.
- The edge of a 6 by 6 tile square is 6.

# Key terms

### prime factor

A prime number that appears in a product equaling a whole number. ($84 = 2^2 \times 3 \times 7$)

### greatest common factor

The largest whole number that divides each number in a set. (GCF)

### least common multiple

The smallest positive whole number that is a multiple of each number in a set. (LCM)

### square root

The non-negative number which multiplies by itself to give n, when n is a perfect square in this lesson. ($\sqrt{n}$)

# Key terms

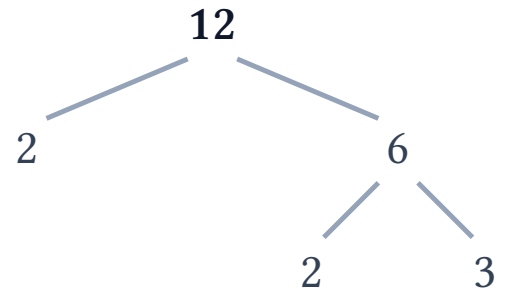
**cube root**

The number which multiplies by itself twice more to give n, when n is a perfect cube in this lesson. ($\sqrt[3]{n}$)

# Factoring 12 and 18

- Start with $12 \div 2 = 6$, then $6 \div 2 = 3$, then $3 \div 3 = 1$.
- The prime factorization of 12 is $2^2 \times 3$.
- Start with $18 \div 2 = 9$, then $9 \div 3 = 3$, then $3 \div 3 = 1$.
- The prime factorization of 18 is $2 \times 3^2$.

**Factor tree for 12**




$12 = 2^2 \times 3$

# GCF of 12 and 18

- The prime factorization of 12 is $12 = 2^2 \times 3$.
- The prime factorization of 18 is $18 = 2 \times 3^2$.
- The lowest shared powers are $2^1$ and $3^1$.
- The GCF is $2 \times 3 = 6$.

**GCF from prime lists**

$12 = 2^2 \times 3$

$18 = 2 \times 3^2$

$\text{GCF} = 2 \times 3 = 6$

# LCM of 12 and 18

- The highest power of 2 is $2^2$.
- The highest power of 3 is $3^2$.
- The LCM is $2^2 \times 3^2 = 36$.

**LCM from prime lists**

Highest 2 power: $2^2$

Highest 3 power: $3^2$

LCM = $4 \times 9 = 36$

# Why 0 and 1 have no prime factors

- The only positive factor of 1 is 1, and 1 sits outside the primes.
- Every product of primes is 2 or larger, so 0 has no prime factors.

**0 and 1**

**1 has only factor 1 (not prime)**

**0 has every integer factor**

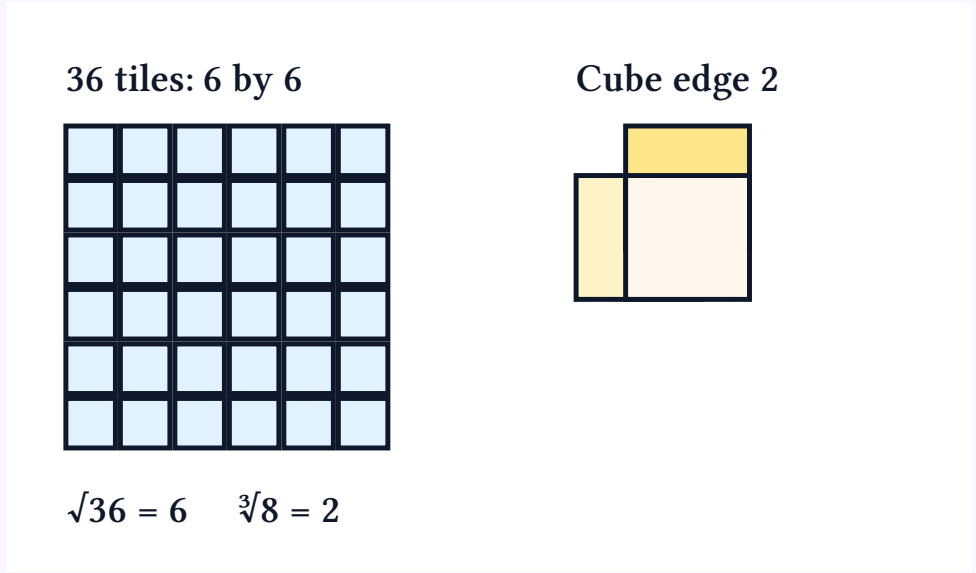
Prime factors start at 2

# GCF and LCM of 84 and 90

- The factorization of 84 is $2^2 \times 3 \times 7$.
- The factorization of 90 is $2 \times 3^2 \times 5$.
- The GCF is $2 \times 3 = 6$.
- The LCM is $2^2 \times 3^2 \times 5 \times 7 = 1260$.

# A square of 36 tiles

- Thirty-six tiles form a 6 by 6 square.
- The square root of 36 is 6, and $6 \times 6 = 36$.

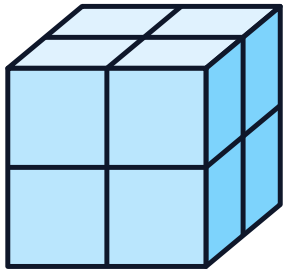


# A cube of 8 and a leftover 50

- Eight unit cubes form a $2 \times 2 \times 2$ cube, so the cube root is 2.
- Fifty sits past $7^2 = 49$ and short of $8^2 = 64$.
- Fifty also sits past $3^3 = 27$ and short of $4^3 = 64$.

**$2 \times 2 \times 2$ cube**

8 unit cubes



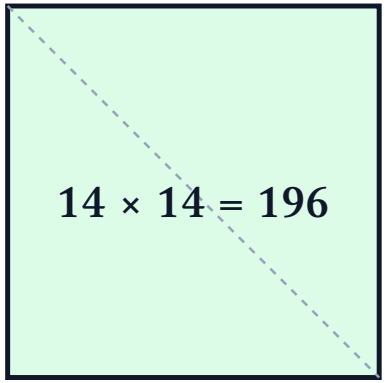
**$\sqrt[3]{8} = 2$, since $2^3 = 8$**

Each edge is 2 units long

# A patio of 196 stones

- The product 14 × 14 = 196.
- The square root of 196 is 14.

**14 by 14 patio**




**√196 = 14**

# A crate of 216 boxes

- The product $6 \times 6 \times 6 = 216$.
- The cube root of 216 is 6.

## Quick check

- The GCF of 60 and 42 is $2 \times 3 = 6$.
- Twenty-seven unit cubes form a $3 \times 3 \times 3$ cube, so the cube root is 3.

# Prime factorization

- The GCF of 12 and 18 is 6.
- The square root of 36 is the side 6.

Mathematics 10C

## Unit AN: Algebra and number

# Irrational numbers

# The diagonal of a unit square

- A 1 by 1 square has diagonal $\sqrt{2}$.
- The length $\sqrt{2}$ is about 1.414, between 1 and 2.

# The real number system

- The fraction ¾ is rational.
- The number $\sqrt{2}$ is irrational, and its index is 2.

# Key terms

### rational number

A number that can be written as a ratio of two integers with a nonzero denominator. ($p/q$)

### irrational number

A real number whose decimal continues without a repeating block, such as $\sqrt{2}$ or $\pi$. ($\sqrt{2}$, $\pi$)

### mixed radical

A radical with a coefficient outside the root sign and a radicand that has no perfect-power factor left. ($6\sqrt{2}$)

### entire radical

A radical whose coefficient is 1, with every factor inside the root sign. ($\sqrt{72}$)

# Key terms

**index**

The small number on a radical that tells how many equal factors are sought. An unwritten index means 2. (the n in $\sqrt[n]{a}$)

# Ordering four numbers

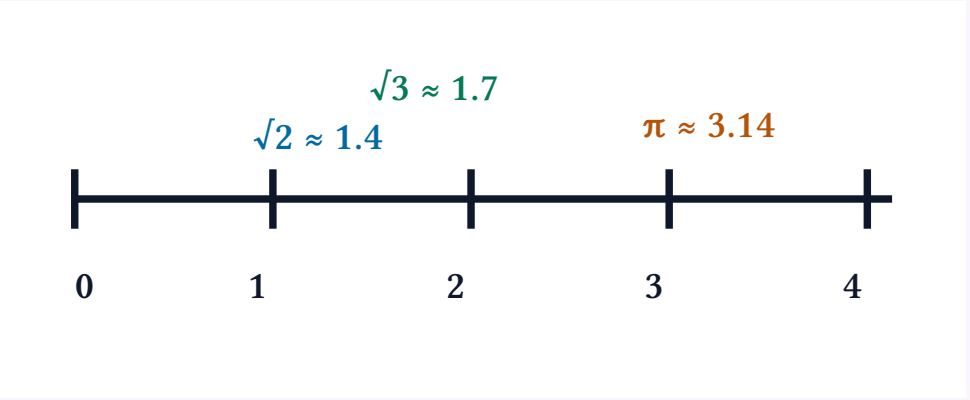
- The number 0.5 equals ½, so it is rational.
- The number $\sqrt{9}$ equals 3, so it is rational.
- The number $\sqrt{2}$ is irrational.
- The number $\pi$ is irrational.

**Order on a number line**



# Placing them on a line

- The value $\sqrt{2}$ is about 1.414.
- The value $\sqrt{3}$ is about 1.732.
- The value $\pi$ is about 3.14, just past 3.



# Simplifying √72

- The radical √72 equals √(36 × 2).
- The root √36 = 6, so √72 = 6√2.
- A calculator shows 6√2 is about 8.485.

# The index of a radical

- The symbol $\sqrt{16}$ means the square root, index 2.
- The symbol $\sqrt[3]{8}$ means the cube root, index 3.
- The roots are $\sqrt{16} = 4$ and $\sqrt[3]{8} = 2$.

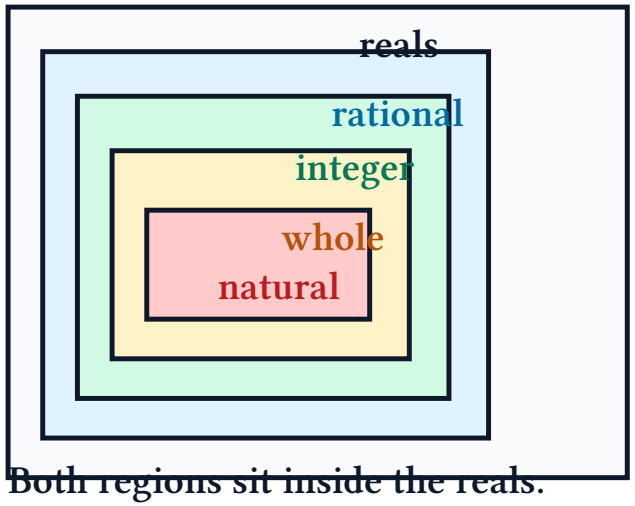
**Index tells the root**

$\sqrt{16}$ index 2 $\rightarrow$ 4

$\sqrt[3]{8}$ index 3 $\rightarrow$ 2

# Where the sets sit

- The natural numbers in this course are 1, 2, 3, and so on.
- Whole numbers are those numbers together with 0.
- Integers add negatives such as −3. Rationals add ratios such as ¾.



# Order a mixed list

- The list is ½, √2, 2, π.
- In order: ½, then √2, then 2, then π.

## Quick check

- Simplify $\sqrt{50}$ as $5\sqrt{2}$.
- Place $\sqrt{2}$ to the left of 2 on the number line.

# A root written two ways

- Keep $6\sqrt{2}$ as the exact simplified form.
- Use about 8.485 when you need a place on the line.

Mathematics 10C

## Unit AN: Algebra and number

# Integral and rational exponents

# Halving a power of 2

- The value $2^3 = 8$.
- The value $2^2 = 4$.
- Each step down divides the value by 2.

# Powers and roots

- In $2^3$, the base is 2 and the exponent is 3.
- The rational exponent $\frac{1}{3}$ on 8 means the cube root.

# Key terms

**power**

A product of equal factors, written with a base and an exponent. ($a^n$)

**base**

The repeated factor in a power. (the a in $a^n$)

**exponent**

The number that tells how the base is repeated, inverted, or rooted. (the n in $a^n$)

**integral exponent**

An exponent that is an integer. A negative integral exponent names a reciprocal power when the base is nonzero. ($a^{-n} = 1/a^n$)

# Key terms

**rational exponent**

An exponent that is a ratio of integers. The unit-fraction exponent $1/n$ names the nth root when n is a positive integer. ($a^{1/n}$)

# Continue the halving row

- Divide $2^1 = 2$ by 2 to get $2^0 = 1$.
- Divide $2^0 = 1$ by 2 to get $2^{-1} = \frac{1}{2}$.
- Divide $2^{-1} = \frac{1}{2}$ by 2 to get $2^{-2} = \frac{1}{4}$.
- Divide $2^{-2} = \frac{1}{4}$ by 2 to get $2^{-3} = \frac{1}{8}$.

| Power | Value |
|---|---|
| $2^3$ | 8 |
| ↓ ÷ 2 | |
| $2^2$ | 4 |
| ↓ ÷ 2 | |
| $2^1$ | 2 |
| ↓ ÷ 2 | |
| $2^0$ | 1 |
| ↓ ÷ 2 | |
| $2^{-1}$ | $\frac{1}{2}$ |
| ↓ ÷ 2 | |
| $2^{-2}$ | $\frac{1}{4}$ |
| ↓ ÷ 2 | |
| $2^{-3}$ | $\frac{1}{8}$ |

# A base of 0

- The power $0^2 = 0$.
- The pattern for a negative exponent asks for a reciprocal.
- The reciprocal of 0 is undefined, so $0^{-2}$ stays undefined.

**Powers of 0**

$0^2 = 0$

$0^{-n}$ needs a reciprocal (undefined here)

# A unit-fraction exponent

- The value $8^{1/3}$ asks for the cube root of 8.
- The product $2 \times 2 \times 2 = 8$, so $8^{1\,/\,3} = 2$.

# Multiplying powers with the same base

- The product $(2^3)(2^4)$ counts $3 + 4$ factors of 2.
- The sum of the exponents is 7.
- The value $2^7 = 128$.

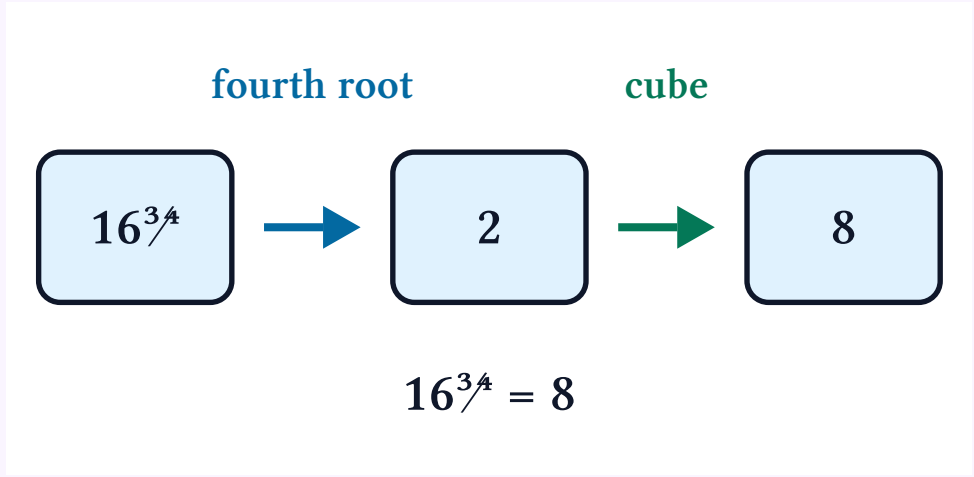
$(2^3)(2^4)$

3 + 4 factors of 2

$2^7$

# Sixteen to the power ¾

- The exponent ¾ means a fourth root, then a cube.
- The fourth root of 16 is 2, because $2^4 = 16$.
- Then $2^3 = 8$, so $16^{3/4} = 8$.



# An added-exponent slip

- The line $(x^2)^3 = x^5$ adds 2 and 3.
- The power-of-a-power law multiplies $2 \times 3$.
- The corrected power is $x^6$.

**$(x^2)^3$**

Wrong: add exponents ⟶ $x^5$

**Correct: multiply $2 \times 3 = x^6$**

# A variable product

- The power $(2x^3)^2 = 4x^6$.
- Then $4x^6 \times x^{-4} = 4x^2$ when x is nonzero.

## Quick check

- The value $4^{-2} = 1/16$.
- The value $27^{2/3} = 9$, because the cube root of 27 is 3 and $3^2 = 9$.

# A power ready for a rectangle

- The value $2^{-3} = \frac{1}{8}$.
- The value $16^{3/4} = 8$.

Mathematics 10C

## Unit AN: Algebra and number

# Multiplying polynomials

# A rectangle with algebra sides

- The width is $x + 3$.
- The height is $x + 2$.
- The area splits into four regions.

# Terms and products

- The expression $x + 3$ has two terms.
- The product $(x + 2)(x + 3)$ multiplies every term by every term.

# Key terms

**monomial**

A polynomial with one term, a coefficient times powers of the variables. ($3x^2$)

**binomial**

A polynomial with two terms. ($x + 4$)

**trinomial**

A polynomial with three terms. ($x^2 + 5x + 6$)

**like terms**

Terms whose variable parts match, so their coefficients can be added. ($5x$ and $2x$)

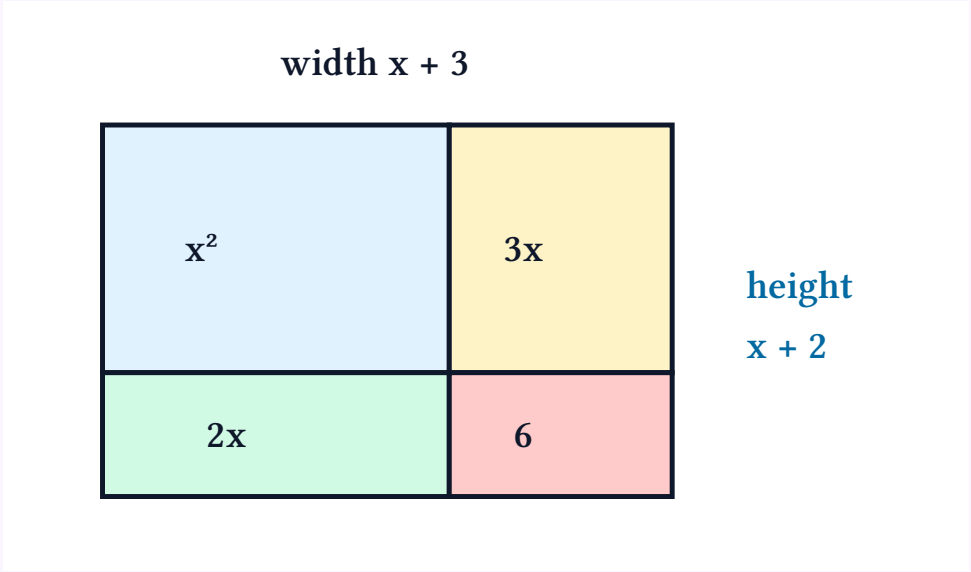
# Key terms

**area model**

A rectangle whose side lengths are the factors and whose inner rectangles are the terms of the product. ((x + 2)(x + 3))

# Four cells for (x + 2)(x + 3)

- The x by x cell is $x^2$.
- The x by 3 cell is 3x, and the 2 by x cell is 2x.
- The 2 by 3 cell is 6.



# Combine the like terms

- The products are $x^2$, 3x, 2x, and 6.
- The sum 3x + 2x is 5x.
- The product is $x^2 + 5x + 6$.

**Products $x^2$, 3x, 2x, 6**

**$x^2 + 5x + 6$**

3x + 2x = 5x

## The number cousin 23 × 14

- Write 23 as 20 + 3 and 14 as 10 + 4.
- The four products are 200, 80, 30, and 12.
- The sum is 322.

# Every pair in $(2x + 1)$(x + 4)

- The four products are $2x^2$, 8x, x, and 4.
- The like terms 8x and x make 9x.
- The product is $2x^2 + 9x + 4$.

**(2x+1)(x+4)**

$2x^2$, 8x, x, 4

**$2x^2 + 9x + 4$**

# Checking by substituting 3

- The factors at x = 3 are 7 and 7, and 7 × 7 = 49.
- The expanded form 2(9) + 9(3) + 4 = 18 + 27 + 4 = 49.

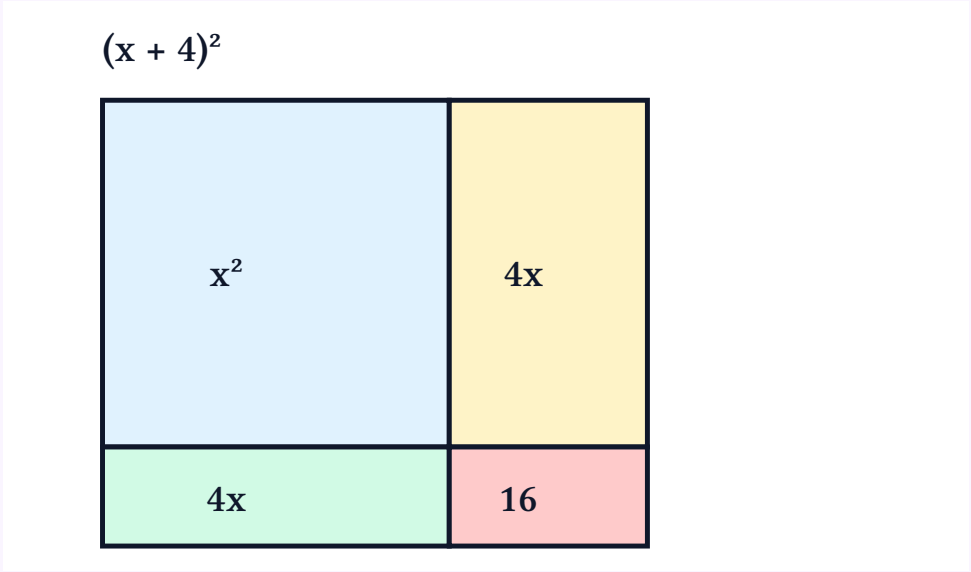
**Substitute x = 3**

**(7)(7) = 49**

**Expanded form also 49**

# The square $(x + 4)^2$

- The four products are $x^2$, 4x, 4x, and 16.
- The like terms make 8x.
- The product is $x^2 + 8x + 16$.



# A longer product

- Multiply $(x + 1)$ by $x^2$, by $2x$, and by $3$.
- The products are $x^3$, $2x^2$, $3x$, $x^2$, $2x$, and $3$.
- The sum is $x^3 + 3x^2 + 5x + 3$.

## Quick check

- The product $(3x + 2)(x + 5)$ is $3x^2 + 17x + 10$.
- The slip $x^2 + 16$ for $(x + 4)^2$ drops the cross terms $4x$ and $4x$.

# Checking a product

- The product $(x + 2)(x + 3)$ is $x^2 + 5x + 6$.
- Factoring will look for the sides that rebuild that trinomial.

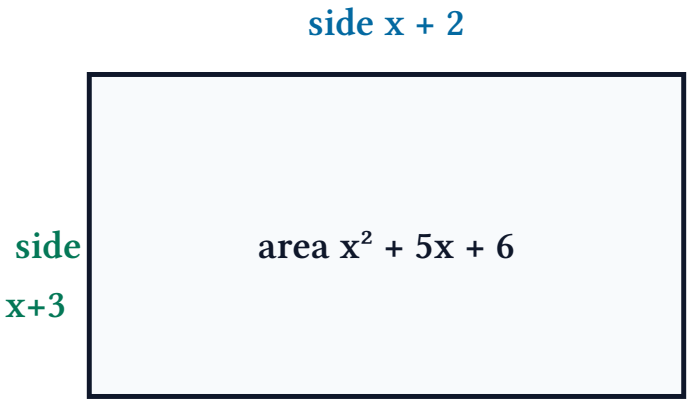
Mathematics 10C

## Unit AN: Algebra and number

# Factoring polynomials

# Factoring trinomials

- The area $x^2 + 5x + 6$ came from sides $x + 2$ and $x + 3$.
- Factoring hunts for those sides.




Factors read from the sides: $(x + 2)(x + 3)$

# Shared factors and rectangles

- The binomial $6x^2 + 9x$ has a common factor $3x$.
- The trinomial $x^2 + 5x + 6$ factors as a rectangle of binomial sides.

# Key terms

### common factor

A factor shared by every term of a polynomial, written outside the brackets. (3x)

### factored form

A polynomial written as a product of polynomials. (3x(2$x$ + 3))

### difference of squares

A binomial whose terms are squares separated by subtraction, the trinomial case with middle coefficient 0. ($x^2$ - 16)

### trinomial

A polynomial with three terms, factored here by a pair of numbers. ($x^2$ + 5x + 6)

# A common factor first

- The terms $6x^2$ and $9x$ share $3x$.
- The factored form is $3x(2x + 3)$.

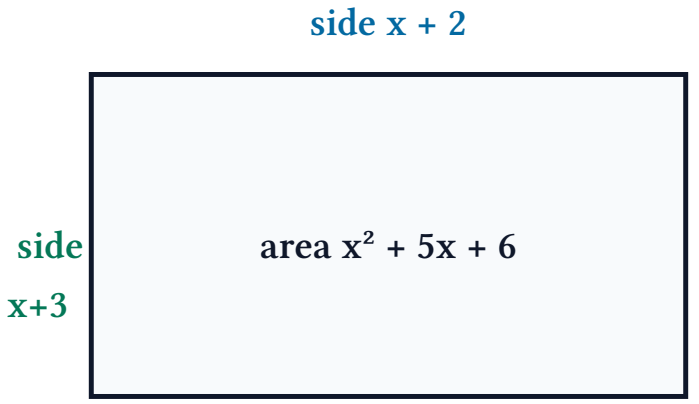
**Factor $6x^2 + 9x$**

**GCF: $3x$**

**$3x(2x + 3)$**

# Two numbers for $x^2 + 5x + 6$

- The pair needs sum 5 and product 6.
- The numbers 2 and 3 work, because $2 + 3 = 5$ and $2 \times 3 = 6$.
- The factors are $(x + 2)(x + 3)$.



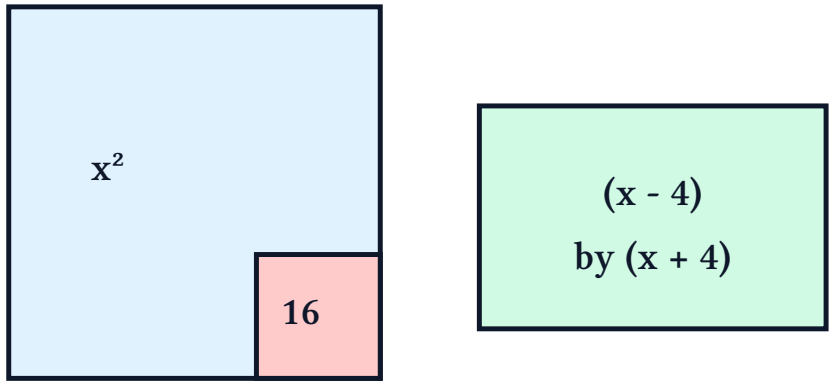

Factors read from the sides: $(x + 2)(x + 3)$

# Checking by multiplication

- The product $(x + 2)(x + 3)$ is $x^2 + 3x + 2x + 6$.
- The like terms make $x^2 + 5x + 6$.

# A difference of squares

- The polynomial $x^2 - 16$ is $x^2 + 0x - 16$.
- The pair $-4$ and $4$ has sum $0$ and product $-16$.
- The factors are $(x - 4)(x + 4)$.




$x^2 - 16$ equals that rectangle

# A pair that fails

- The claim uses $(x + 1)(x + 6)$ for $x^2 + 5x + 6$.
- The sum $1 + 6$ is 7, and the product is 6.
- The expansion is $x^2 + 7x + 6$, a different middle term.

**Check $(x + 1)(x + 6)$**

**Sum: $1 + 6 = 7$**

**Need sum 5 for $x^2 + 5x + 6$**

Use factors with product 6 and sum 5

**$2 \times 3 = 6$, $2 + 3 = 5$: $(x + 2)(x + 3)$**

# Factoring $x^2 - 9$

- The middle coefficient is 0.
- The pair $-3$ and 3 has product $-9$.
- The factors are $(x - 3)(x + 3)$.

## Quick check

- The polynomial $4x^2 - 10x$ factors as $2x(2x - 5)$.
- The polynomial $x^2 - 16$ factors as $(x - 4)(x + 4)$.

# Checking a factorization

- The factors of $x^2 + 5x + 6$ are $(x + 2)(x + 3)$.
- Multiplying those sides returns the trinomial.

Mathematics 10C

## Unit RF: Relations and functions

# Graphs of situations

# A bus ride in time

- The bus waits 2 minutes.
- It drives for 6 minutes, then waits again.
- Time keeps moving between the notes.

# Inputs and outputs

- In the pair (2, 7), the input is 2 and the output is 7.
- A joined segment means the story continues between the dots.

# Key terms

**domain**

The set of input values that the graph, table, or ordered pairs actually uses. (x-values)

**range**

The set of output values that correspond to those inputs. (y-values)

**discrete graph**

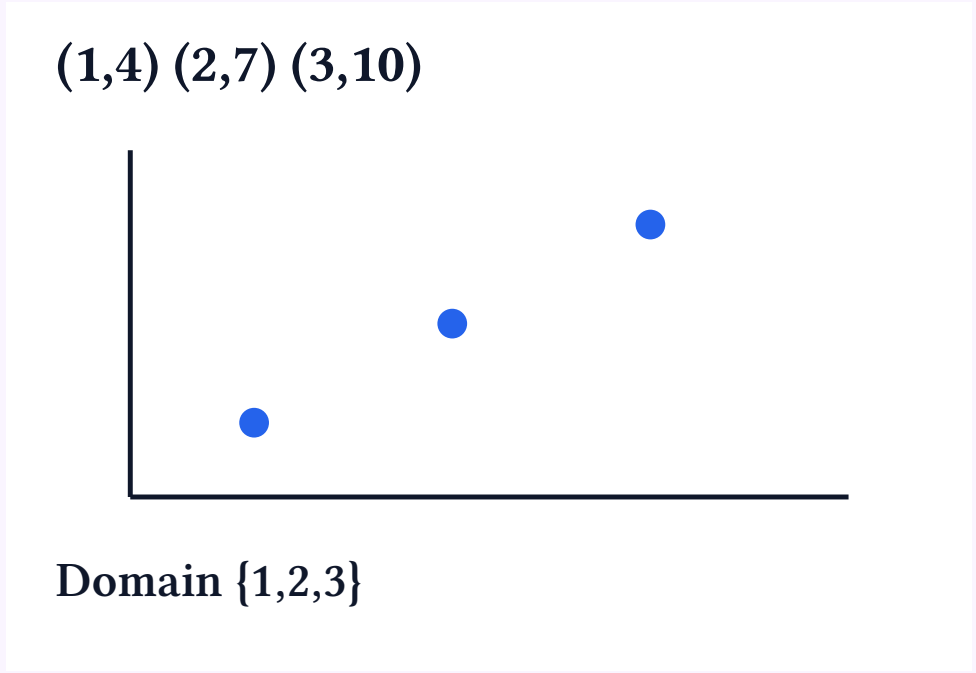
A graph of separate points, used when each point is its own count. (separate points)

**continuous graph**

A graph with the points joined, used when the quantity exists between the recorded values. (a joined line)

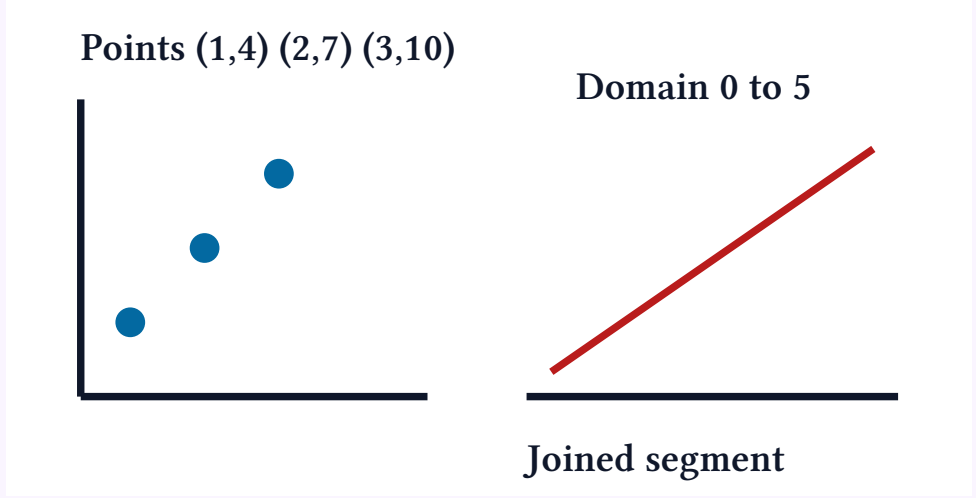
# Domain and range of three points

- The pairs are (1, 4), (2, 7), and (3, 10).
- The domain is {1, 2, 3}.
- The range is {4, 7, 10}.



# A segment from x = 0 to x = 5

- The segment runs from x = 0 to x = 5.
- The domain is $0 \le x \le 5$.
- If the heights run from 1 to 4, the range is $1 \le y \le 4$.

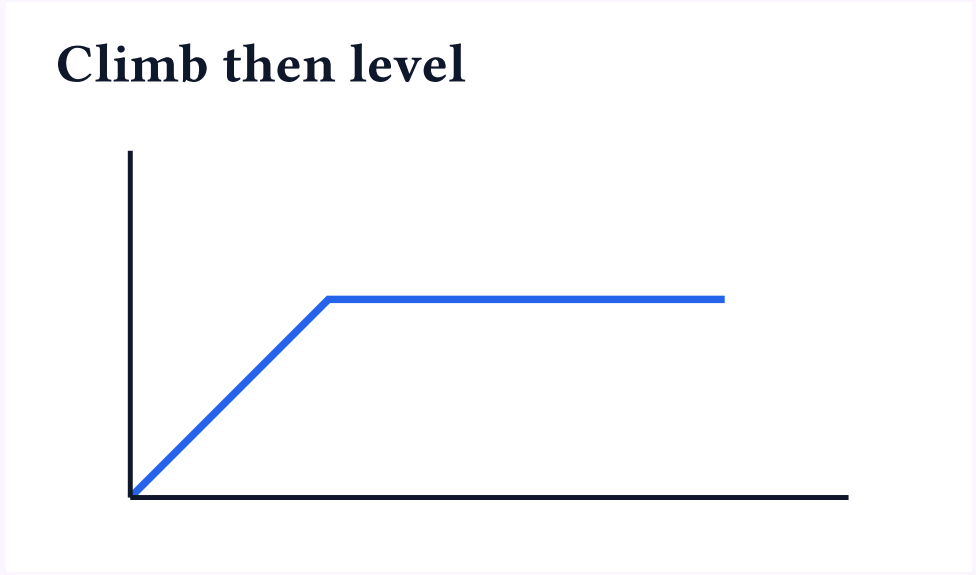


# A fare table and a 12 km trip

- The table uses inputs 1, 2, and 3, so the domain is $\{1, 2, 3\}$.
- A trip from 0 km to 12 km has domain $0 \le d \le 12$.
- If the graph height runs from 0 to 12, the range is $0 \le y \le 12$.

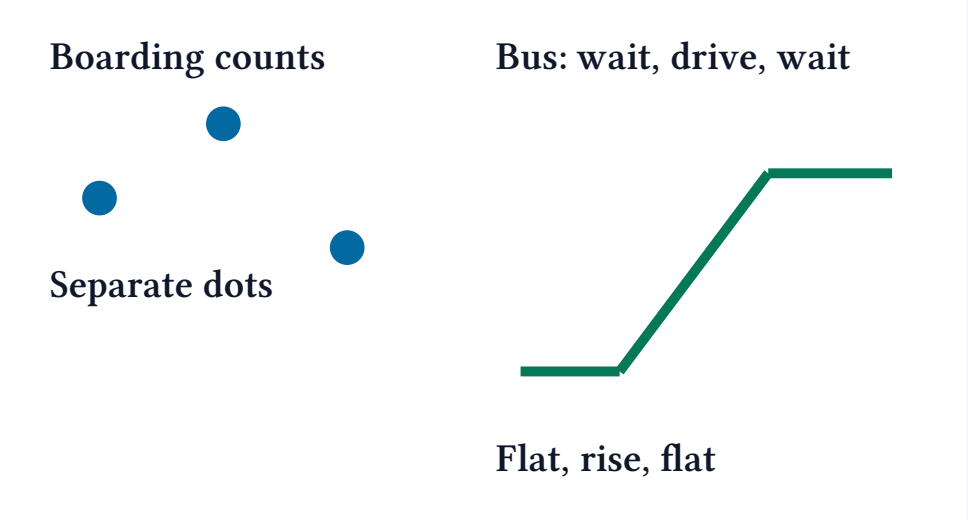
# Join the points when the story continues

- From $x = 0$ to $x = 4$ the output climbs from 0 to 8.
- From $x = 4$ to $x = 7$ the output stays 8.
- The domain is $0 \le x \le 7$ and the range is $0 \le y \le 8$.



# Dots for separate counts

- Riders boarding at each stop are separate counts.
- Those counts stay as dots.
- A line through the dots would invent riders between stops.



# A bus trip graph

- Draw a flat piece for the first 2 minutes.
- Draw a rise for the next 6 minutes.
- Draw a flat piece for the last wait.

## Quick check

- The pairs $(0, 2)$ and $(4, 2)$ have domain $\{0, 4\}$ and range $\{2\}$.
- A continuing walk between those times uses domain $0 \le t \le 4$.

# Identifying a set

- Name the domain before you describe the shape.
- Join points only when the story continues between them.

Mathematics 10C

## Unit RF: Relations and functions

# Relations and functions

# One locker, one combination

- Locker 14 has one combination.
- A sign-up lists Alex in room 4 and Alex in room 9.
- The same name has two rooms.

# One output for each input

- The pairs (1, 4) and (2, 4) give input 2 a single output.
- A function pairs each input with exactly one output.

# Key terms

**relation**

A set of ordered pairs, shown as a list, a table, or a graph. (a set of ordered pairs)

**function**

A relation in which each input pairs with exactly one output. (one output for each input)

**ordered pair**

A pair that lists an input first and its output second. ((x, y))

**vertical line test**

A check that slides a vertical line across a graph and expects a single meeting point. (one meeting point)

# A repeated input

- The set is {(1, 4), (1, 9), (3, 7)}.
- Input 1 has outputs 4 and 9.
- This relation stays outside the functions.

Each input once

| input | output |
|---|---|
| 1 | 4 |
| 2 | 4 |
| 3 | 7 |

Input 1 twice

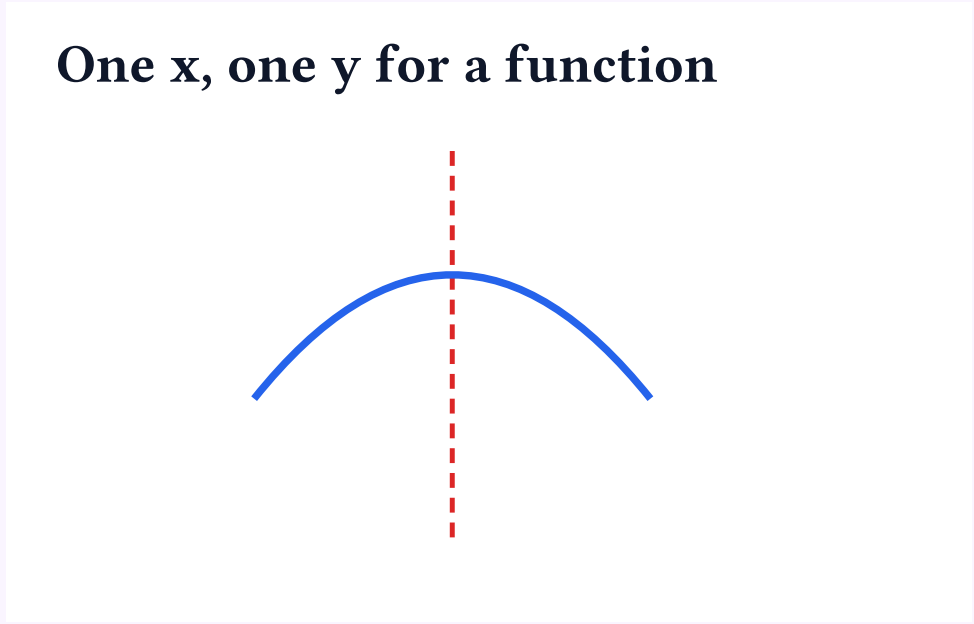
| input | output |
|---|---|
| 1 | 4 |
| 1 | 9 |
| 3 | 7 |

## A set that passes

- The set $\{(0, 1), (2, 1)\}$ has inputs 0 and 2.
- Each input appears once.
- The set is a function, and the shared output 1 is allowed.

# A vertical line on a graph

- A vertical line through one x meets a function graph once.
- A graph shaped like a C opening to the right meets that line twice.
- The C-shaped graph stays outside the functions.



# Rules in words

- Each locker number pairs with one combination.
- That sentence is the function rule, with the graph put away.

**One locker → one combo**

**Function rule in words**

# Four examples

- Set A = {(0, 1), (2, 1)} is a function.
- Set B = {(3, 4), (3, 5)} stays outside the functions.
- A horizontal line is a function. A C opening right stays outside.

## Quick check

- The set {(2, 5), (2, 8), (4, 5)} stays outside the functions.
- A vertical line meets a horizontal graph once, so that graph is a function.

# One output per input

- Check the inputs before you check the outputs.
- A repeated input with two outputs stays outside the functions.

Mathematics 10C

## Unit RF: Relations and functions

# Slope of a line

# A ramp and a staircase

- The ramp rises 1 m over a 4 m run.
- The staircase rises 1 m over a 1 m run.
- The staircase is steeper.

# Rise over run

- Slope is rise divided by run.
- A line through (1, 2) and (4, 8) has slope 2.

# Key terms

**slope**

The ratio of vertical change to horizontal change along a line. (m = rise/run)

**rise**

The vertical change between two points, positive when the line goes up. (change in y)

**run**

The horizontal change between two points, positive when moving to the right. (change in x)

**rate of change**

A slope that tells how much one quantity changes for each unit of another. (dollars per kilometre)

# Key terms

**parallel lines**

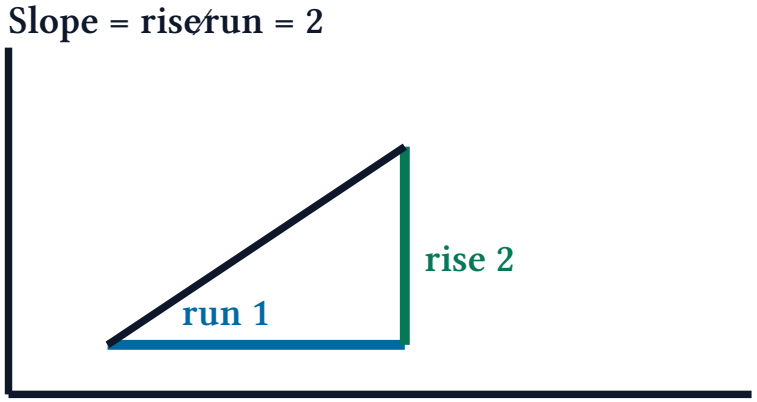
Lines that share one slope and stay the same distance apart forever. (equal slopes)

**perpendicular lines**

Lines that meet at a right angle. For two sloping lines, the product of the slopes is $-1$. (slopes multiply to $-1$)

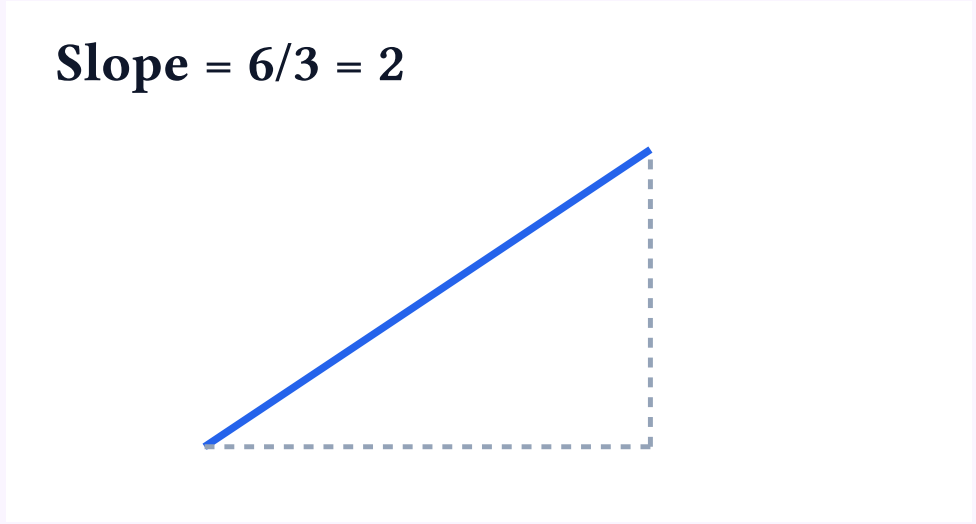
# Slope from (1, 2) to (4, 8)

- The rise is $8 - 2 = 6$.
- The run is $4 - 1 = 3$.
- The slope is $\frac{6}{3} = 2$.



# The same slope farther along

- From (4, 8) to (7, 14), the rise is 6.
- The run is 3.
- The slope is again 2.

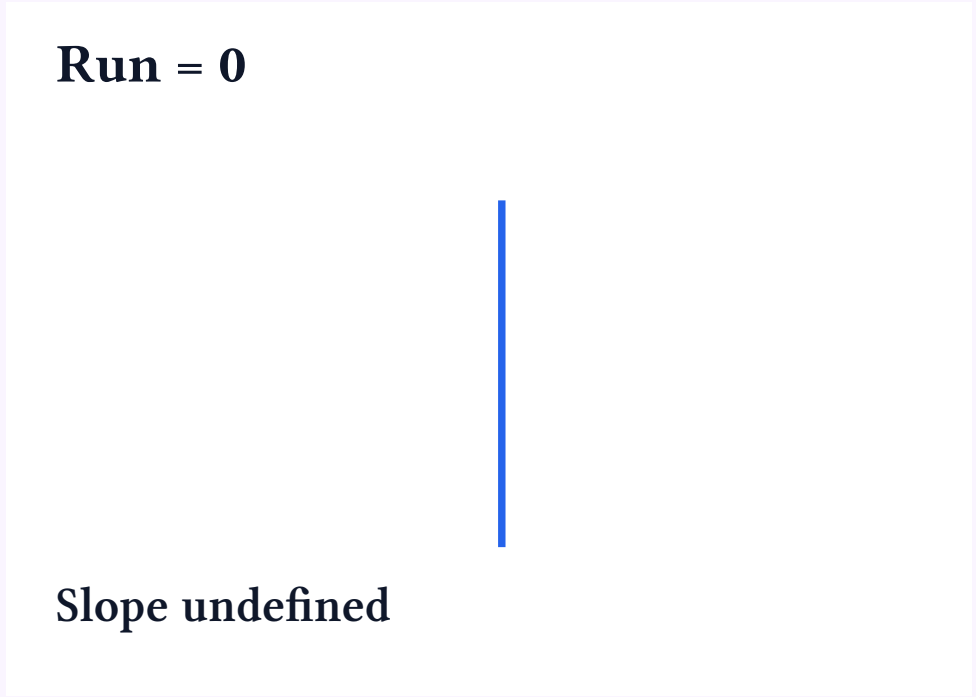


## A falling line and a flat line

- From $(0, 5)$ to $(2, 1)$, the rise is $1 - 5 = -4$.
- The run is $2$, so the slope is $-2$.
- A horizontal line from $(0, 2)$ to $(6, 2)$ has rise $0$ and slope $0$.

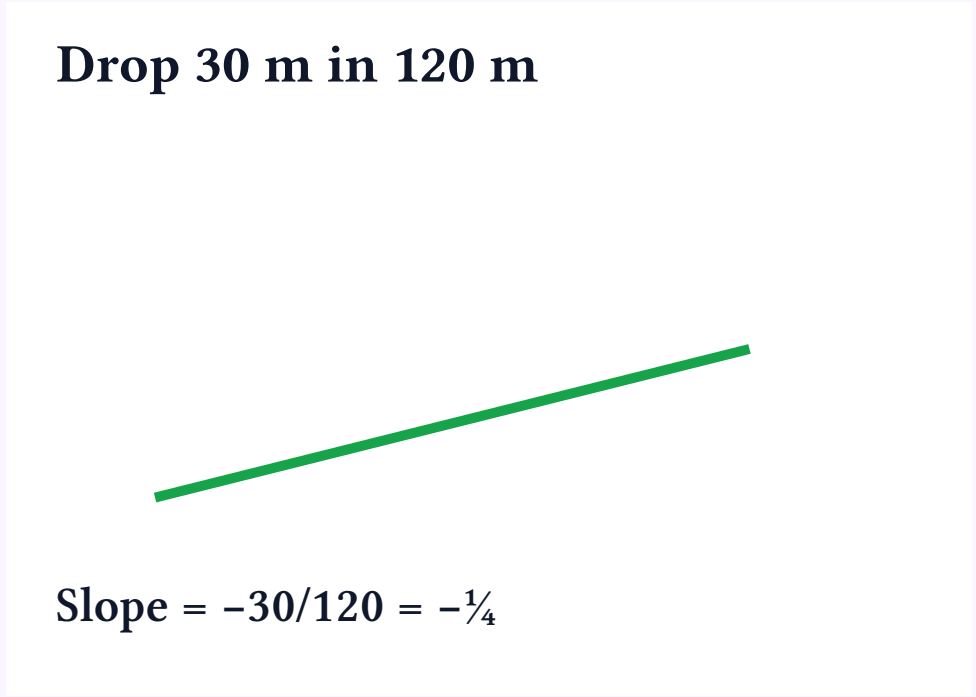
# A vertical line

- The points (3, 4) and (3, 9) share an x-coordinate.
- The run is $3 - 3 = 0$.
- The slope is undefined, and the line is vertical.



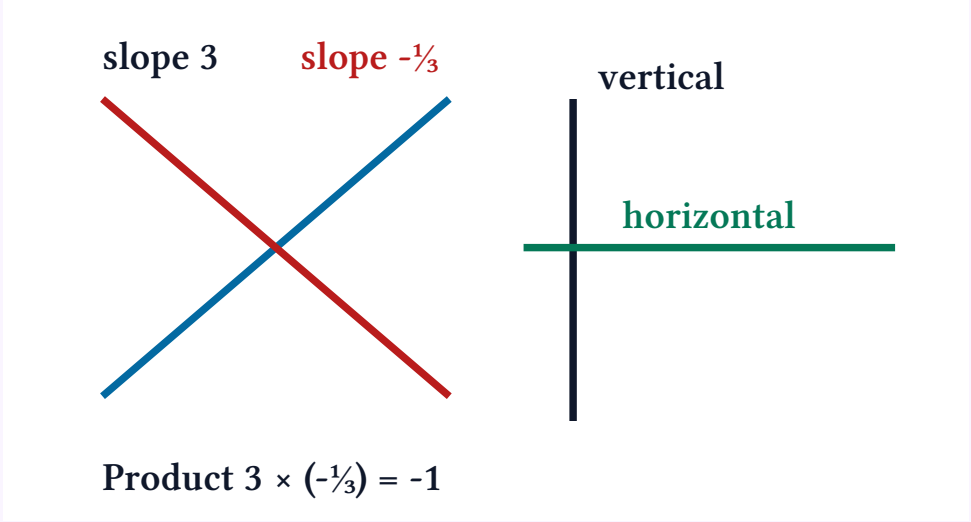
# A trail as a rate

- The trail drops 30 m over a run of 120 m.
- The slope is $-\frac{30}{120} = -\frac{1}{4}$.
- That rate is 1 m of drop for every 4 m of run.



# A new point and a right angle

- From (2, 1) with slope 3, one run and three rises land on (3, 4).
- A parallel line also has slope 3.
- A perpendicular slope is $-\frac{1}{3}$, because $3 \times (-\frac{1}{3}) = -1$.



## A hill of rise 15

- The rise is 15 m and the run is 45 m.
- The slope is $^{15}\!/_{45} = \frac{1}{3}$.

## Quick check

- A drop of 8 m over a run of 24 m has slope $-\frac{8}{24} = -\frac{1}{3}$.
- The slope perpendicular to 2 is $-\frac{1}{2}$, because $2 \times \left(-\frac{1}{2}\right) = -1$.

## Slope as a ratio

- Slope 2 means rise 2 for run 1.
- A run of 0 leaves the slope undefined.

Mathematics 10C

## Unit RF: Relations and functions

# Representing linear relations

# A taxi fare in two lines

- The flag drop is $4.
- Each kilometre adds $2.
- At 0 km the fare is $4, and at 1 km it is $6.

# Five versions of one fare

- The equation is $C = 2d + 4$.
- The graph through $(0, 4)$ and $(1, 6)$ is a straight line.

# Key terms

### linear relation

A relation whose rate of change is constant and whose graph is a straight line.
(y = 2x + 4)

### independent variable

The input you choose, written in the first coordinate or on the horizontal axis. (x or d)

### dependent variable

The output that responds to the input, written in the second coordinate or on the vertical axis. (y or C)

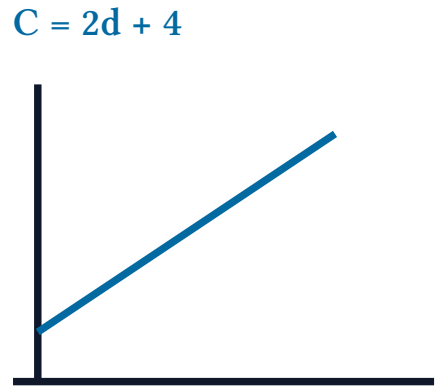
### first difference

The change in consecutive outputs when the inputs increase by the same step. (Δy)

# Three representations

- The fares 4, 6, 8, 10 rise by 2 each kilometre.
- Those equal jumps make a straight line.
- The equation is C = 2d + 4.

| km | cost |
|---|---|
| 0 | $4 |
| 1 | $6 |
| 2 | $8 |



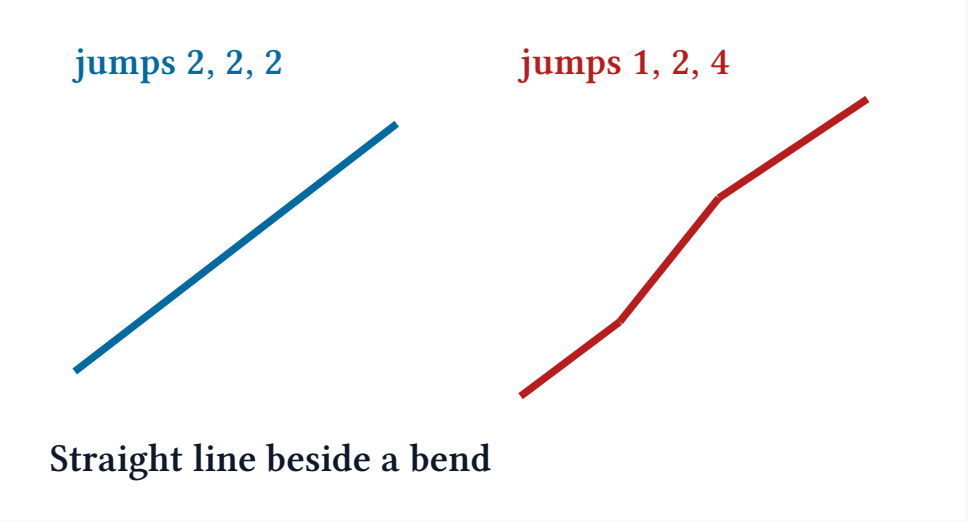

Table, line, and equation agree

## Testing the point (3, 10)

- In the story, 3 km costs $4 + 2 \times 3 = 10$ dollars.
- The pair (3, 10) is on the line.
- The equation $C = 2(3) + 4$ also gives 10.

# Equal jumps or changing jumps

- The outputs 1, 3, 5, 7 have differences 2, 2, and 2, so the relation is linear.
- The outputs 1, 2, 4, 8 have differences 1, then 2, then 4, so the relation is nonlinear.



## Four representations

- A curve is nonlinear.
- The equation $y = -x + 6$ is linear.
- Outputs 10, 7, 4, 1 have differences −3, −3, and −3, so that table is linear.
- The area of a square as the side grows is nonlinear.

## Quick check

- The pair (2, 8) fits C = 2d + 4, because 2(2) + 4 = 8.
- Outputs 2, 5, 10 are nonlinear because the jumps are 3 and 5.

# Displays that agree

- The fare C = 2d + 4 matches the points (0, 4) and (1, 6).
- A changing jump means the graph bends.

Mathematics 10C

## Unit RF: Relations and functions

# Characteristics of linear graphs

# Where the line meets the axes

- The line passes through (0, 4) and (2, 0).
- It meets the y-axis at 4 and the x-axis at 2.

# Features of a linear graph

- The slope between $(0, 4)$ and $(2, 0)$ is $(0 - 4)/(2 - 0) = -2$.
- A full line uses every real number as an input and as an output.

# Key terms

**x-intercept**

The point where a graph meets the horizontal axis, so the output is 0. ((a, 0))

**y-intercept**

The point where a graph meets the vertical axis, so the input is 0. ((0, b))

**domain**

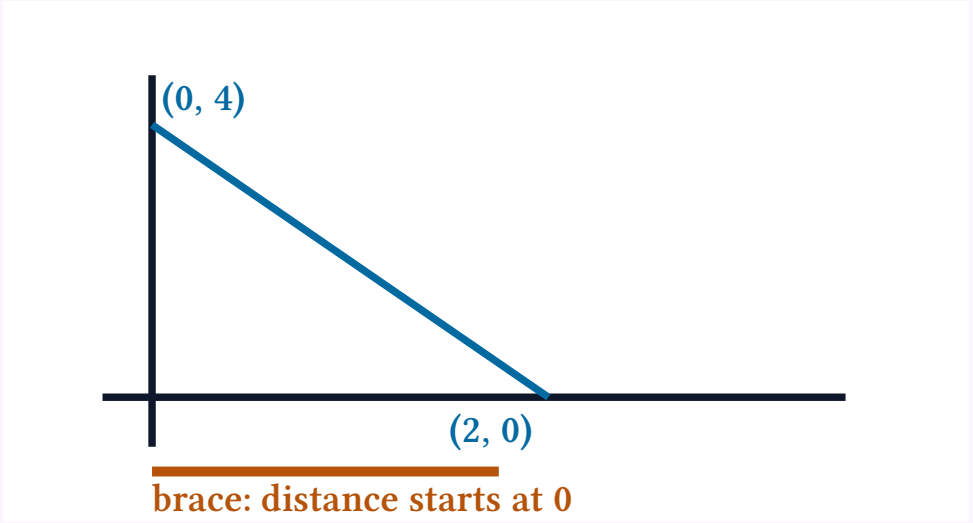
The set of inputs the relation uses, all real numbers for a full non-vertical line. (allowed x)

**range**

The set of outputs the relation uses, all real numbers for a full non-horizontal line. (allowed y)

# Four features of the full line

- The y-intercept is (0, 4).
- The x-intercept is (2, 0).
- The slope is −2, and the domain and range are all real numbers.

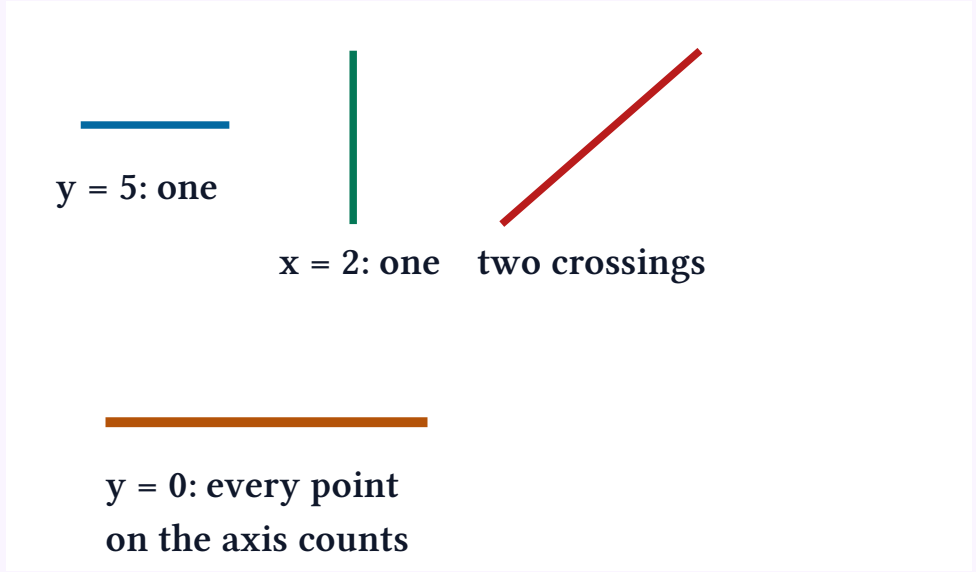


# A phone bill

- The bill is $C = 0.10m + 20$ for $m \geq 0$.
- The y-intercept is (0, 20) and the slope is 0.10.
- The domain is $m \geq 0$ and the range is $C \geq 20$.

# How many intercepts

- The line $y = x + 1$ meets the axes at $(0, 1)$ and $(-1, 0)$.
- The horizontal line $y = 3$ meets the y-axis once, at $(0, 3)$.
- The x-axis, $y = 0$, has every point on it as an x-intercept.



# A ray of the fare

- With $x \geq 0$ and $y \geq 0$, the graph runs from $(0, 3)$ to $(3, 0)$.
- The domain is $0 \leq x \leq 3$ and the range is $0 \leq y \leq 3$.

## Quick check

- The line through $(0, -1)$ and $(2, 3)$ has slope $2$ and y-intercept $-1$.
- The bill $C = 0.10m + 20$ with $m \geq 0$ has range $C \geq 20$.

# Reading four features

- Name the intercepts, then the slope.
- Then say whether a story cuts the domain or the range.

Mathematics 10C

## Unit RF: Relations and functions

# Forms of a line

## One line, three equations

- The equation y = $-3x + 6$ has slope −3 and y-intercept 6.
- The equation $3x + y - 6 = 0$ rearranges to that same line.
- The equation y − 6 = −3(x − 0) is the same line again.

## Three forms

- Slope-intercept form shows m and b, as in $y = -3x + 6$.
- General form sets one side to 0, as in $3x + y - 6 = 0$.
- Slope-point form starts from a known point, as in $y - 6 = -3(x - 0)$.

# Key terms

### slope-intercept form

A linear equation that shows the slope m and the y-intercept b. (y = mx + b)

### general form

A linear equation written as a sum of terms equal to 0. (Ax + By + C = 0)

### slope-point form

A linear equation built from the slope and one point on the line. (y - y1 = m(x - x1))

# Two slope-point versions

- Using $(0, 6)$: $y - 6 = -3(x - 0)$.
- Using $(2, 0)$: $y - 0 = -3(x - 2)$.
- Both simplify to $y = -3x + 6$.

**Slope −3**

$y - 6 = -3(x - 0)$

$y = -3(x - 2)$

# From general form to slope and intercept

- Start with $2x - 4y + 8 = 0$.
- Then $4y = 2x + 8$, so $y = (\frac{1}{2})x + 2$.
- The slope is $\frac{1}{2}$ and the y-intercept is 2.

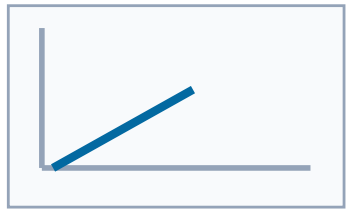
# Graphing from any form

- For $y = (\frac{1}{2})x + 2$, start at (0, 2) and rise 1 across a run of 2.
- For $2x - 4y + 8 = 0$, the intercepts are (−4, 0) and (0, 2).
- For $y - 2 = (\frac{1}{2})(x - 4)$, plot (4, 2) and use the slope.



**Stair**

(0, 2), rise 1 run 2



**Intercepts**

(−4, 0) and (0, 2)

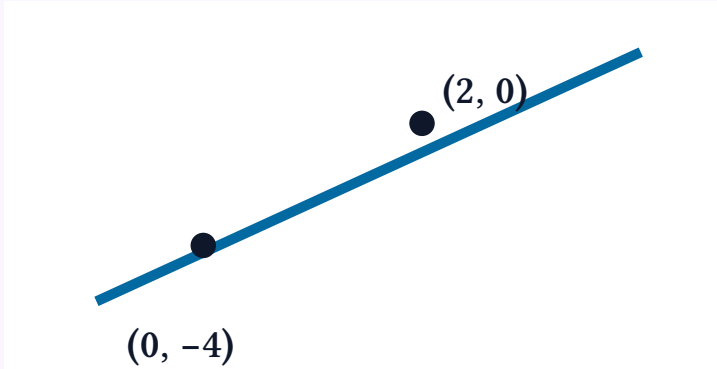


**Point**

(4, 2), slope ½

# Matching three equations

- The set is $y = 2x - 4$.
- The set includes $x - (\frac{1}{2})y - 2 = 0$.
- The set includes $y - 0 = 2(x - 2)$.
- All three simplify to $y = 2x - 4$.



## Rewriting $y - 1 = 4(x - 2)$

- Expand to $y - 1 = 4x - 8$.
- Then $y = 4x - 7$.
- The slope is 4 and the y-intercept is $-7$.

## Quick check

- The equation $3x + y - 6 = 0$ rewrites as y = $-3x + 6$.
- The graph starts at (0, 6) and falls 3 for a run of 1.

## Choosing a form

- Use slope-intercept form when you want m and b.
- Use slope-point form when you already know a point and a slope.

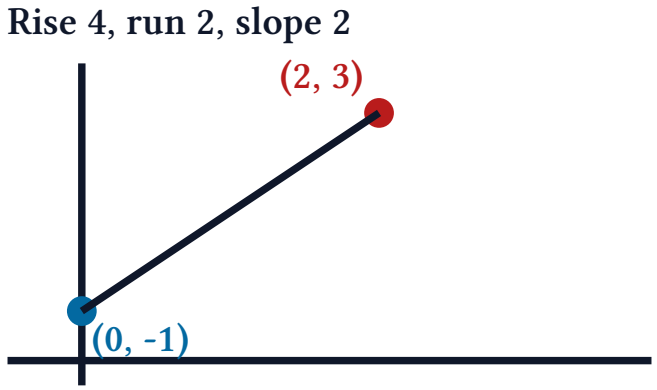
Mathematics 10C

## Unit RF: Relations and functions

# Equations of a line

# Equation from a graph

- The line passes through (0, −1) and (2, 3).
- The rise is 4 and the run is 2, so the slope is 2.
- The equation is $y = 2x - 1$.



## Writing an equation

- From slope 2 and the point (1, 4), write $y - 4 = 2(x - 1)$.
- A parallel line keeps that slope. A perpendicular line uses the slope that multiplies with it to $-1$.

# Key terms

### slope-intercept equation

The equation of a line written so the slope and the y-intercept are the two numbers you can see. (y = mx + b)

### two-point slope

The slope you get by subtracting the coordinates of two points on the line. (m = (y2 - y1)/(x2 - x1))

### parallel slope

The slope you copy from a line when the new line must run in the same direction. (same m)

### perpendicular slope

The slope that multiplies the given slope to make −1, so the lines meet at a right angle. (m1 times m2 = -1)

# Two points on a pay line

- The pay is $50 at 2 hours and $90 at 6 hours.
- The slope is $(90 - 50)/(6 - 2) = 10$ dollars per hour.
- Using $(2, 50)$: $y - 50 = 10(x - 2)$, so $y = 10x + 30$.

# Checking a pay equation

- At x = 2, y = 10 × 2 + 30 = 50.
- At x = 6, y = 10 × 6 + 30 = 90.
- At x = 0, the pay is $30.

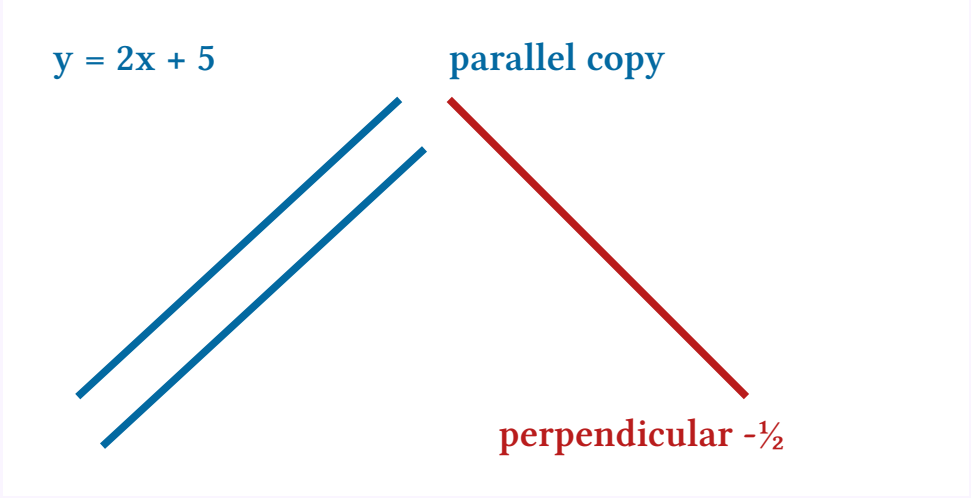
**y = 10x + 30**

**x=2 ⟶ y=50**

**x=6 ⟶ y=90**

# A parallel line through (1, 4)

- The line $y = 2x + 5$ has slope 2.
- The parallel line through (1, 4) keeps slope 2.
- The equation $y - 4 = 2(x - 1)$ simplifies to $y = 2x + 2$.



# A perpendicular line through (1, 4)

- The slope perpendicular to 2 is $-\frac{1}{2}$.
- Check: $2 \times (-\frac{1}{2}) = -1$.
- The equation is $y - 4 = (-\frac{1}{2})(x - 1)$.

**Slope 2 and $-\frac{1}{2}$**

$2 \times (-\frac{1}{2}) = -1$

Through (1, 4)

## Writing and testing

- The line through (0, 1) and (3, 7) has slope 2.
- The equation is $y = 2x + 1$.
- At $x = 3$, $y = 7$.

## Quick check

- A line through $(0, -1)$ with slope $4$ has equation $y = 4x - 1$.
- A line parallel to it through $(2, 3)$ has equation $y - 3 = 4(x - 2)$.

# Testing an equation

- Build the slope, then anchor a point.
- Substitute a known point as a check.

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## Unit RF: Relations and functions

# Function notation

# Substituting −1

- The rule is f(x) = $3x - 2$.
- The input −1 gives f(−1) = 3(−1) − 2 = −5.

# A name for the output

- The rule f(x) = $3x$ — 2 matches y = 3x − 2.
- The symbol g(t) = 7 + t uses t as the input name.

# Key terms

**function notation**

A way to name a function and its output, with the input written in parentheses. (f(x) = $3x - 2$)

**domain value**

A number you are allowed to put into the function. (the input x)

**range value**

The number the function gives back for a domain value. (the output f(x))

# From an input to an output

- The input $-1$ goes into $f(x) = 3x - 2$.
- The calculation is $-3 - 2$.
- The output is $-5$, so the point is $(-1, -5)$.




f(x) = 3x - 2

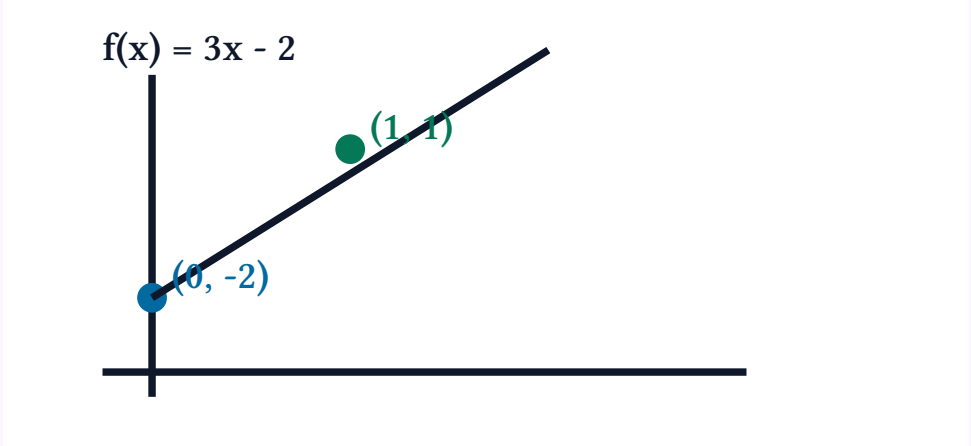
Point on the line: (-1, -5)

# From an output back to the input

- Solve $g(t) = 15$ when $g(t) = 7 + t$.
- The equation $7 + t = 15$ gives $t = 8$.
- The input 8 produces the output 15.

# Sketching f(x) = $3x - 2$

- The y-intercept is (0, −2), because f(0) = −2.
- The slope is 3, so another point is (1, 1).
- Draw the line through (0, −2) and (1, 1).



# Using function notation

- The value $h(2)$ for $h(x) = 5 - 2x$ is $5 - 4 = 1$.
- Solving $h(x) = 1$ gives $5 - 2x = 1$, so $x = 2$.
- In two variables the rule is $y = 5 - 2x$.

## Quick check

- The value f(4) for f(x) = $3x - 2$ is 10.
- Solving g(t) = 15 for g(t) = 7 + t gives t = 8.

# Evaluating a function

- The value $f(-1) = -5$ for $f(x) = 3x - 2$.
- The graph of f is the line $y = 3x - 2$.

Mathematics 10C

## Unit RF: Relations and functions

# Systems of linear equations

## Two lines that meet

- The lines are $y = x + 1$ and $y = -x + 5$.
- At $x = 2$ both give $y = 3$.
- The shared point $(2, 3)$ is the solution of the system.

# A pair that works twice

- A system here is two linear equations.
- The solution of a system is the pair that makes both equations true.

# Key terms

### system of linear equations

Two linear equations considered together, each drawing a line. (two equations)

### solution of a system

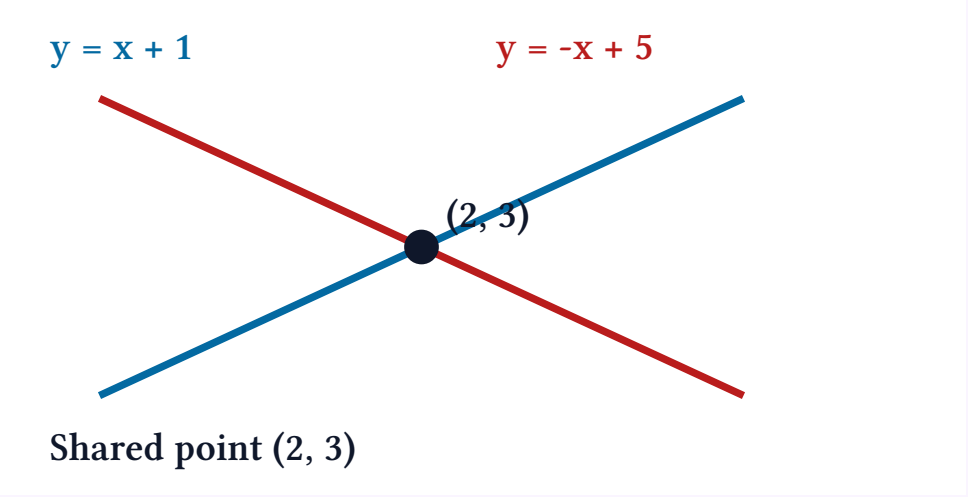
The ordered pair that makes both equations true at the same time. ((x, y))

### point of intersection

The point where the two lines meet, which is the solution you can see on the graph. (the shared point)

# The intersection point

- Set x + 1 equal to −x + 5.
- Then $2x = 4$, so x = 2 and y = 3.
- Check: 2 + 1 = 3 and −2 + 5 = 3.

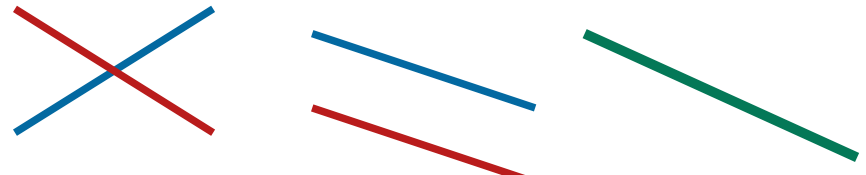


## Add to remove y

- The equations are x + y = 5 and x − y = 1.
- Adding them gives $2x = 6$, so x = 3.
- Then y = 2, and the checks are 3 + 2 = 5 and 3 − 2 = 1.

# How many solutions

- The lines $y = 2x + 1$ and $y = -x + 4$ meet once.
- The lines $y = 2x + 1$ and $y = 2x + 4$ stay apart.
- The equation $2y = 4x + 2$ is the line $y = 2x + 1$ again.



one point lines apart one shared line

Different slopes meet once.

Same slope, new intercept: apart.

# One intersection by algebra

- Set $2x + 1$ equal to −x + 4.
- Then $3x = 3$, so x = 1 and y = 3.
- Check: 2(1) + 1 = 3 and −1 + 4 = 3.

**2x + 1 = −x + 4**

**3x = 3 ⟶ x = 1**

**y = 3**

# Two phone plans

- Plan A is $y = 0.10x + 20$.
- Plan B is $y = 0.20x + 10$.
- Set them equal: $0.10x + 20 = 0.20x + 10$, so $x = 100$ and $y = 30$.

## Quick check

- Solving y = x + 4 and y = $-2x + 10$ gives (2, 6).
- The lines y = 3x + 1 and y = 3x − 5 stay apart.

# A pair that works twice

- The lines $y = x + 1$ and $y = -x + 5$ meet at $(2, 3)$.
- A solution makes both equations true.