Geometry and trigonometry slide deck

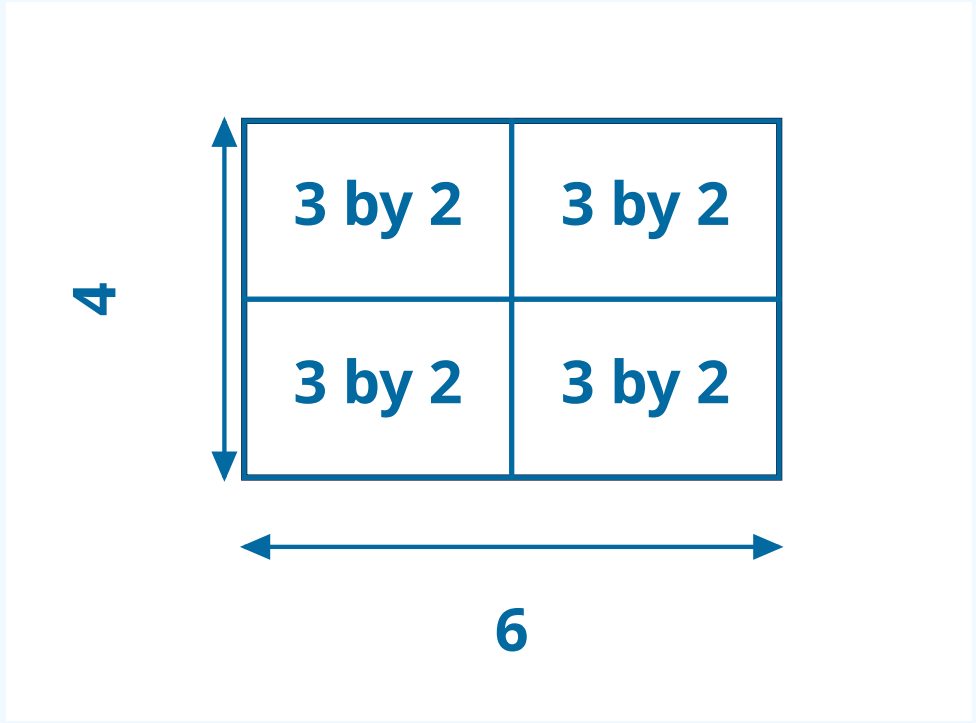
Mathematics 10-3

Unit G: Geometry and trigonometry

Packing puzzles and sliding tiles

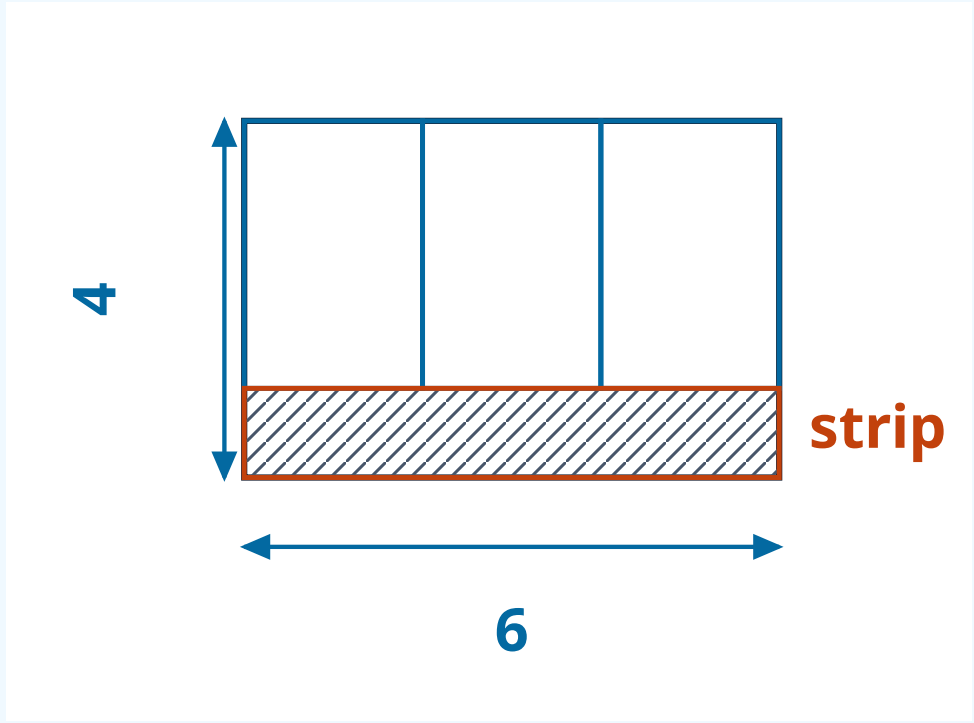
Four panels in a crate

- A 6 by 4 crate has to hold panels that are 3 by 2.
- Two panels fit along the 6, and two fit along the 4.
- That layout uses 4 panels and covers the crate.
- Turn every panel and the 4 side is no longer a whole number of panel edges.



Ways to work a packing puzzle

- Guess and check means you try one layout, then write what happened.
- A systematic list keeps every try.
- A drawing or a model shows the crate.
- Eliminating a try drops a layout that leaves a gap.
- Simplify means you solve a smaller crate first.
- Work backward means you start from a full crate and undo one panel.



Key terms

systematic list

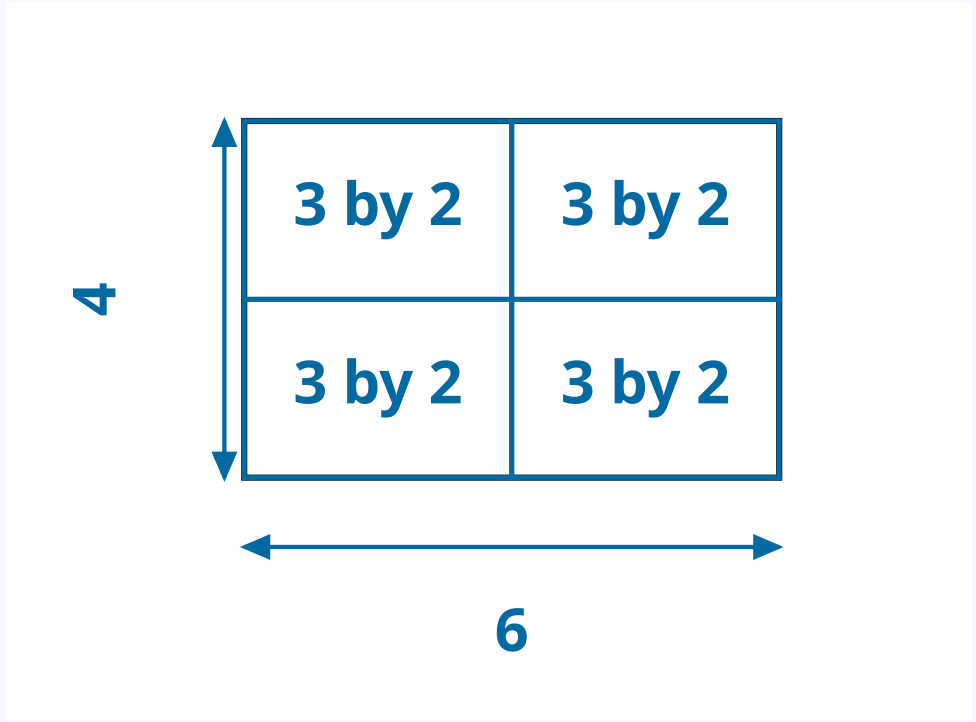
A written record of every try, kept in order, so the next try is a new move.

gap

The empty square in a sliding-tile tray. Tiles move by sliding into that empty square.

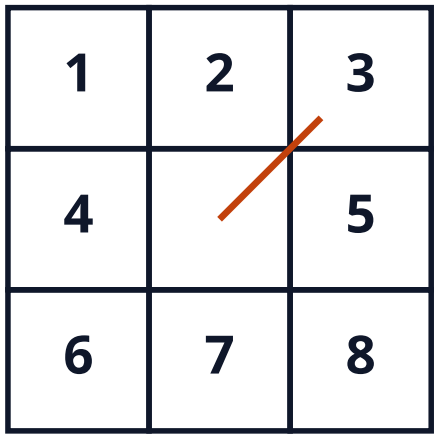
Area count and edge fit

- The crate area is 6 times 4, which is 24.
- Each panel is 3 times 2, which is 6.
- That 24 divided by 6 is 4 panels.
- Along 6 and 4, the panels sit as 3 plus 3 and 2 plus 2.
- That uses the 4 panels and the edges match.
- Turned panels ask 4 divided by 3.
- That split leaves a strip, so this orientation stays off the truck.



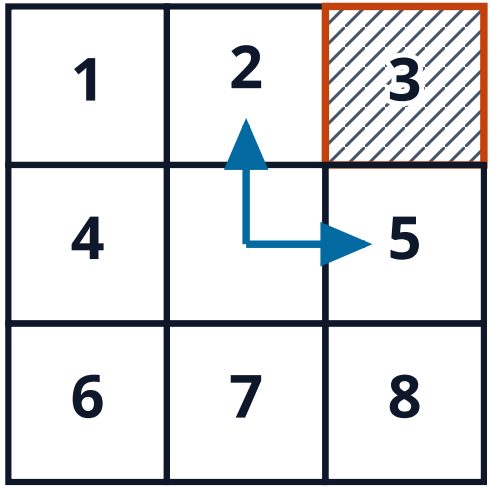
A lane of sliding tiles

- A 3 by 3 tray holds eight numbered tiles and one gap.
- A tile moves by sliding into the gap.
- The gap moves one square up, down, left, or right.
- A diagonal jump skips a square, so the tile would teleport.
- Finish the top row first, then treat the leftover 2 by 3 block as a smaller puzzle.



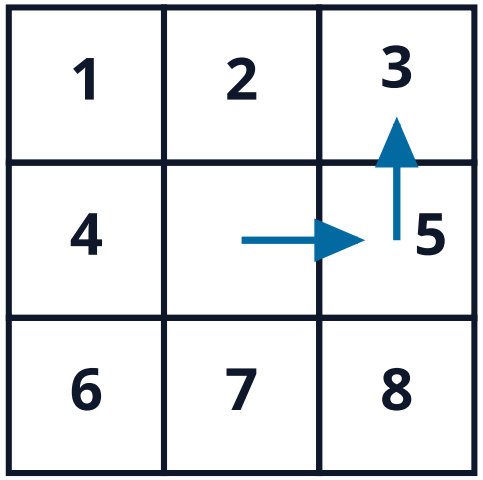
Block a corner and try again

- Cover the top-right square so no tile can land there.
- The goal is still a finished top row of the open squares.
- Work backward from that finished row: the gap has to sit just under the row before the last slide.
- Write the new rule in one sentence so a partner can try your variation.



Correct the written solution

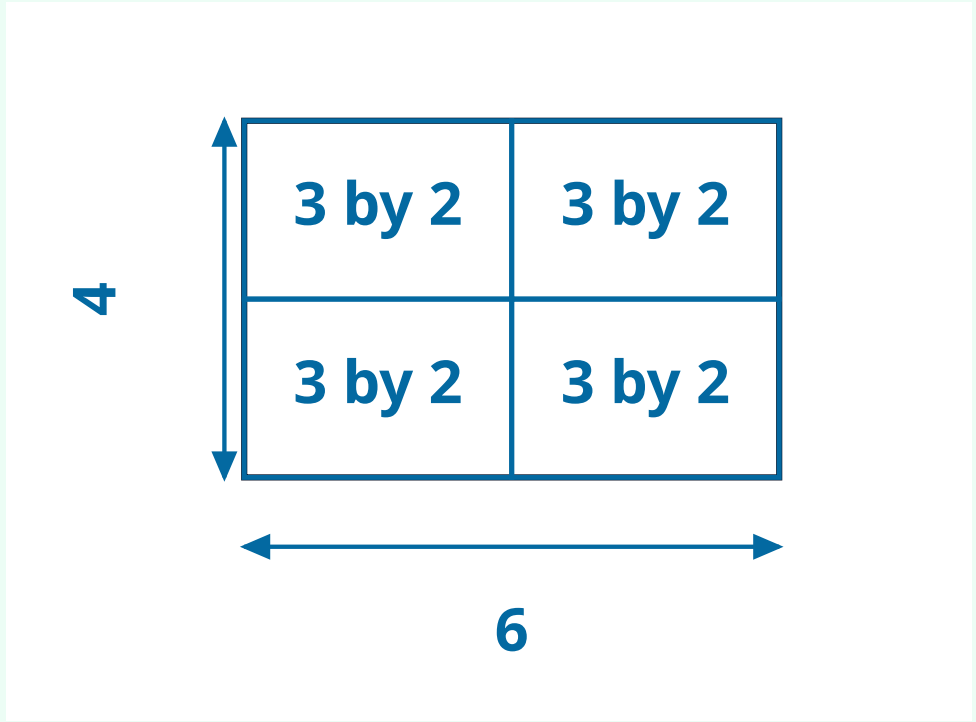
- A classmate's list says the gap moved from the middle square straight to a corner.
- The middle square touches the corner only at a point.
- The gap's next square has to share a full side.
- The fix is two slides: one to the side, then one up or down into the corner.



side, then up

Crate and tray check

- Tell how many 3 by 2 panels cover a 6 by 4 crate, and which way they face.
- Name the error in a list that slides the gap diagonally, and give the two-slide fix.



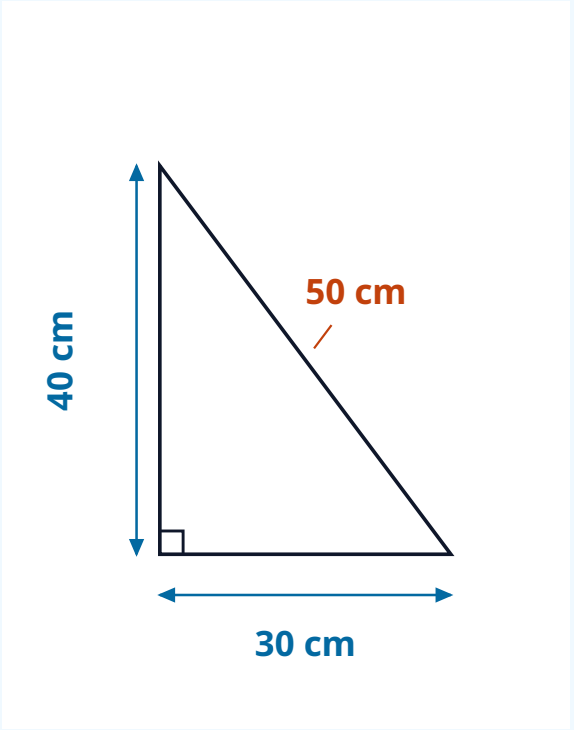
Mathematics 10-3

Unit G: Geometry and trigonometry

Right triangles and square corners

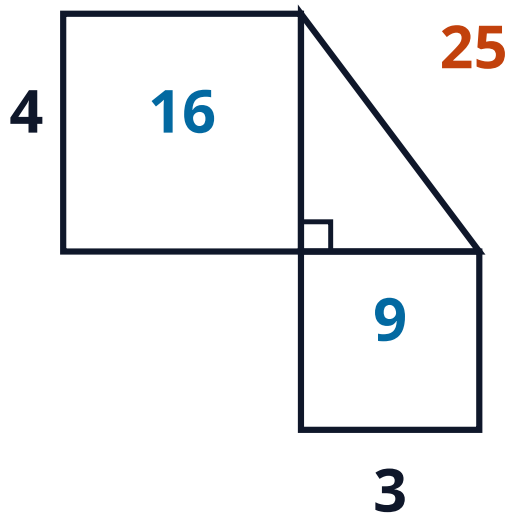
A corner that has to be square

- A framer marks 30 cm along one wall and 40 cm along the next wall.
- The diagonal between those marks should be 50 cm.
- The marks 30, 40, and 50 are 3, 4, and 5 each multiplied by 10.
- Builders have used a 3-4-5 rope to set a square corner, and a welder uses the same check on a frame.



Squares on the three sides

- The hypotenuse is the longest side, across from the right angle.
- Call the shorter sides a and b, and the hypotenuse c.
- The theorem says a squared plus b squared equals c squared.
- Squared means the number times itself.
- A 3-4-5 triangle gives 9 plus 16, which equals 25.
- A 2-3-4 triangle gives 4 plus 9, which is 13, while 4 squared is 16.



Key terms

hypotenuse

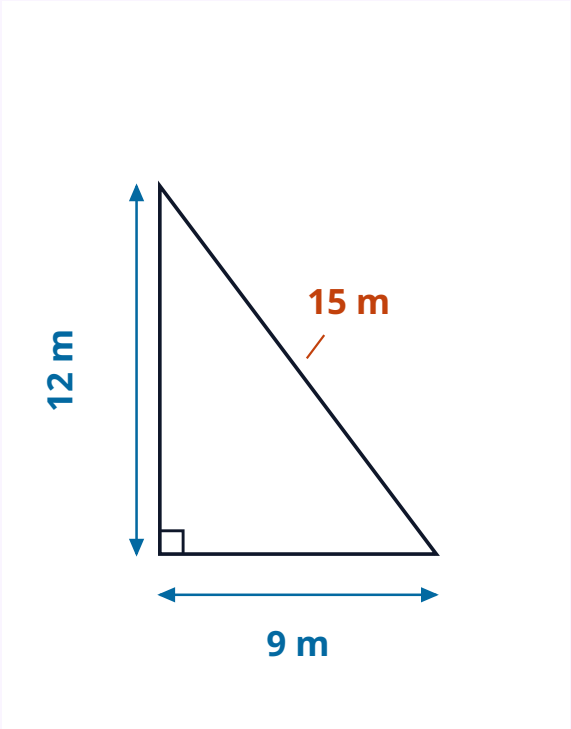
The longest side of a right triangle, the side across from the right angle. (c)

Pythagorean theorem

On a right triangle, the squares of the two shorter sides add to the square of the hypotenuse. ($a^2 + b^2 = c^2$)

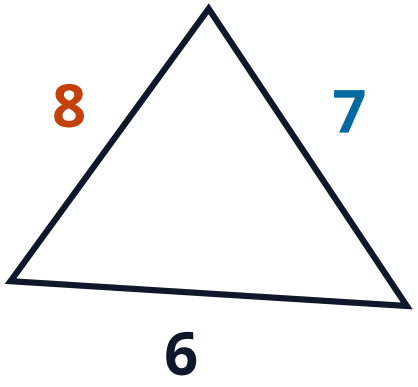
When the squares agree

- If a squared plus b squared equals c squared, the angle opposite c is a right angle.
- Square 9 to get 81 and square 12 to get 144.
- Adding 81 and 144 equals 225.
- Squaring 15 gives 225, so the 9 m by 12 m diagonal is 15 m.
- Square 6 to get 36 and square 8 to get 64.
- Adding 36 and 64 equals 100.
- Squaring 10 gives 100, so the brace on an 8 m run is 10 m.



A triangle that fails the square test

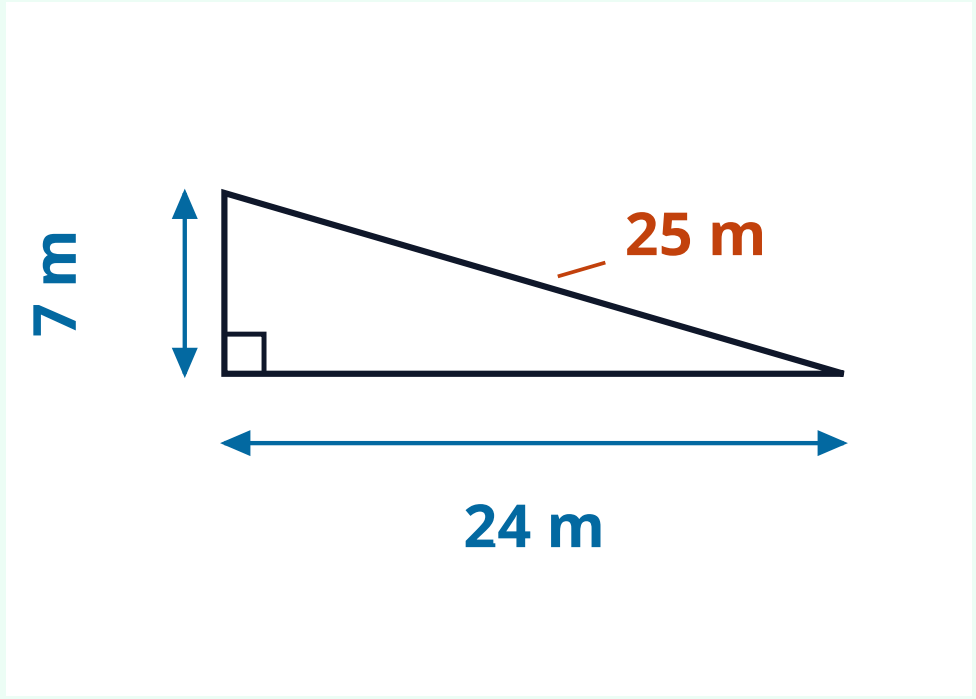
- Sides 6, 7, and 8 ask you to test the longest side, 8.
- Squaring 6 gives 36 and squaring 7 gives 49.
- Those add to 85.
- Squaring 8 gives 64.
- The totals 85 and 64 disagree, so this triangle fails the right-angle test.



$36 + 49 = 85$, and $8^2 = 64$

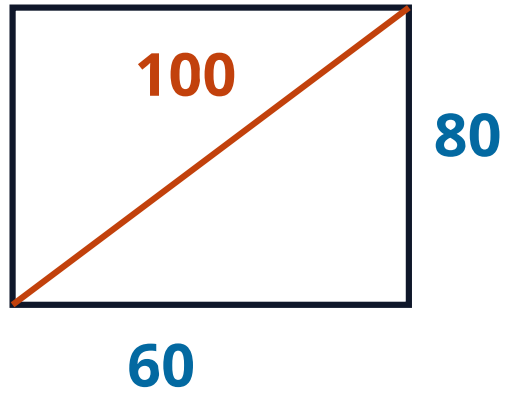
A 7 m by 24 m brace

- A gate opening is 7 m across the base and 24 m up the post.
- You need the diagonal brace.
- Squaring 7 gives 49 and squaring 24 gives 576.
- Those add to 625.
- Squaring 25 gives 625, so the brace is 25 m.



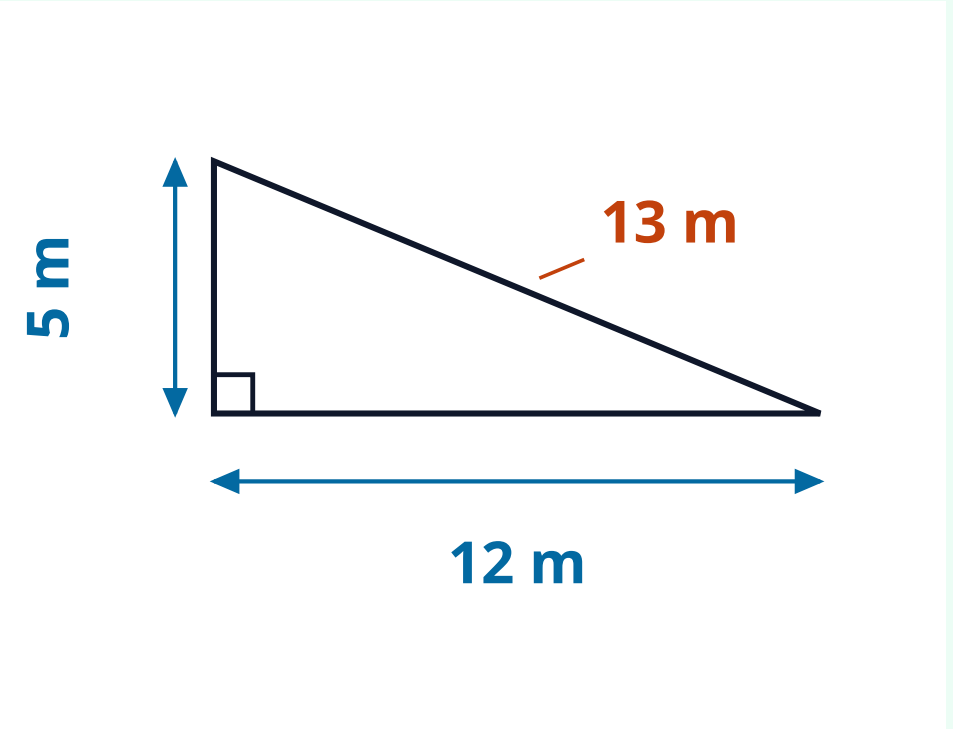
Square a corner on the job

- Multiply 3, 4, and 5 by the same number to get a larger square check.
- Times 10 gives 30, 40, and 50.
- If the measured diagonal matches 50 cm, the corner between the 30 cm and 40 cm marks is a right angle.
- A parallelogram with a 3-4-5 corner and equal diagonals is a rectangle.
- The square corner is what makes the frame true.



Square-corner check

- Find the diagonal of a 9 m by 12 m frame.
- Say what 36 plus 49 versus 64 tells you about a 6-7-8 triangle.



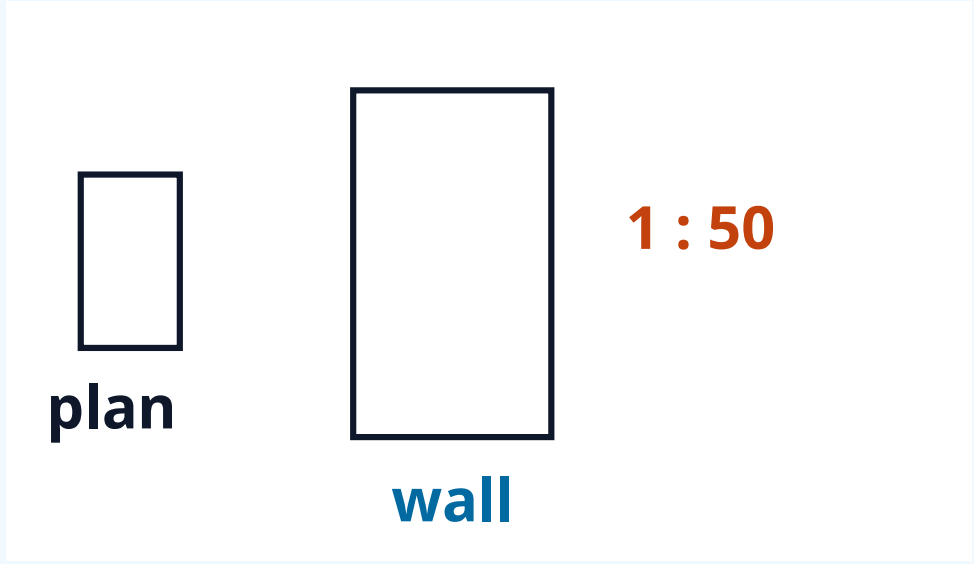
Mathematics 10-3

Unit G: Geometry and trigonometry

Similar polygons and scale drawings

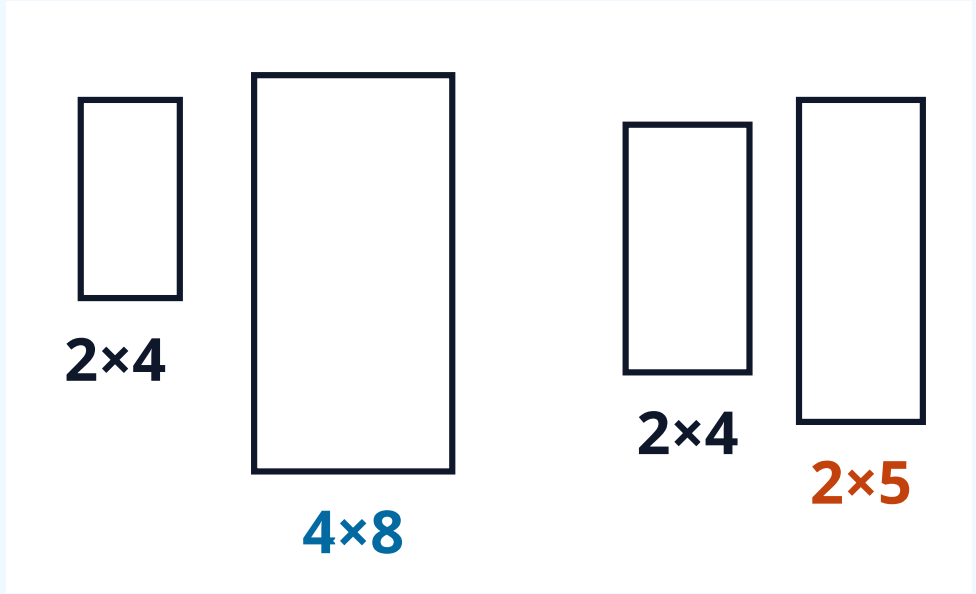
A wall on a site plan

- A site plan uses the scale 1 to 50.
- A wall that is 8.0 metres on the ground is 16 centimetres on the paper.
- The paper shape and the real wall share the same angles.
- Every real length is 50 times the paper length.



Angles match and sides stay in ratio

- Similar polygons have equal matching angles and one constant ratio between matching sides.
- A 2 by 4 rectangle and a 4 by 8 rectangle share right angles, and each side of the larger one is double.
- A 2 by 4 rectangle and a 2 by 5 rectangle share right angles, while the side ratios are 1 and 2.5.
- The similarity test fails.



Key terms

similar polygons

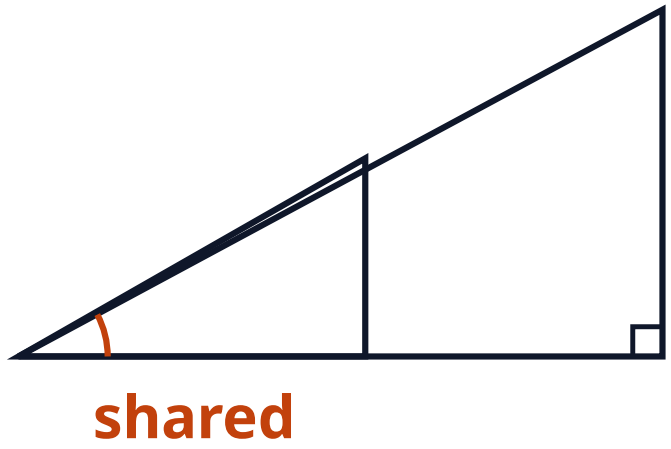
Polygons whose matching angles are equal and whose matching sides stay in one constant ratio.

scale factor

The number you multiply every side by when you draw a larger or smaller copy with the same angles. (1:50)

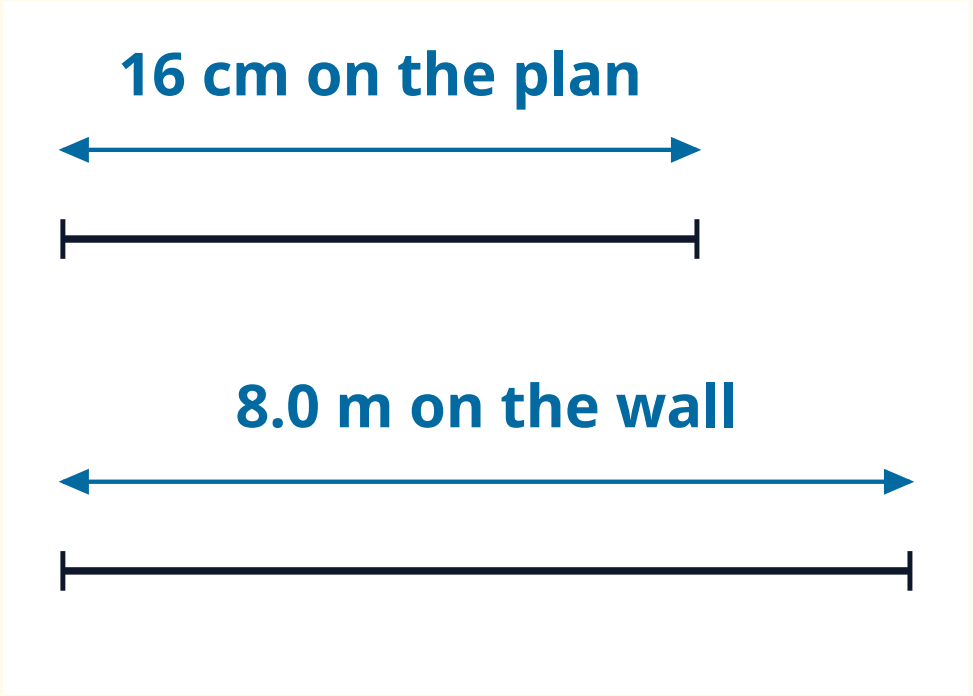
Triangles that share an acute angle

- Two right triangles that share an acute angle also share the right angle, so the third angles match too.
- Matching angles on a triangle force the sides into one ratio.
- A 3-4-5 triangle and a 9-12-15 triangle have ratio 3.
- On a polygon with more than three sides, matching angles still need a side-ratio check.
- Rectangles show why.



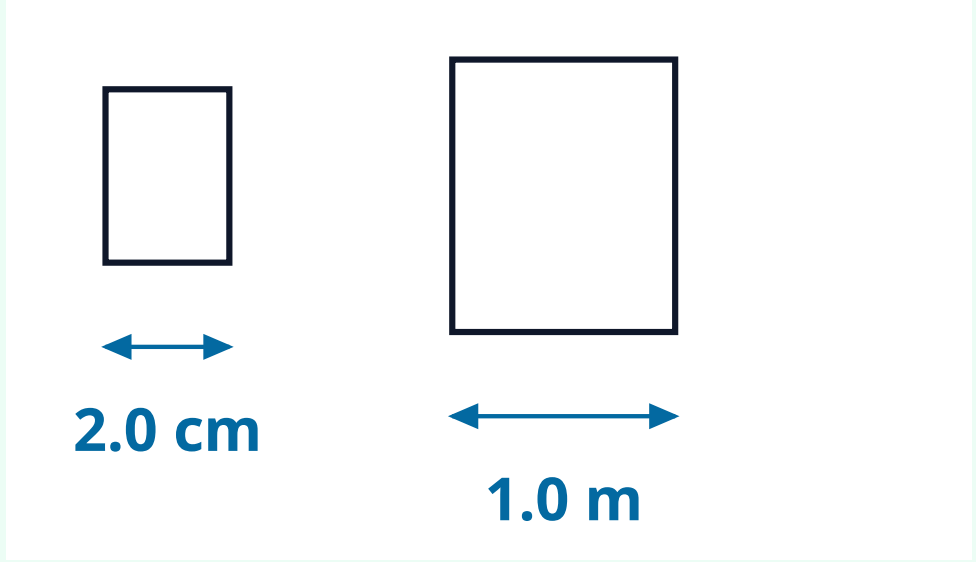
Wall length from a plan

- The scale 1 to 50 means 1 centimetre on the plan stands for 50 centimetres on the ground.
- The wall is 8.0 metres, which is 800 centimetres.
- Dividing 800 by 50 gives 16, so the wall is 16 centimetres on the plan.



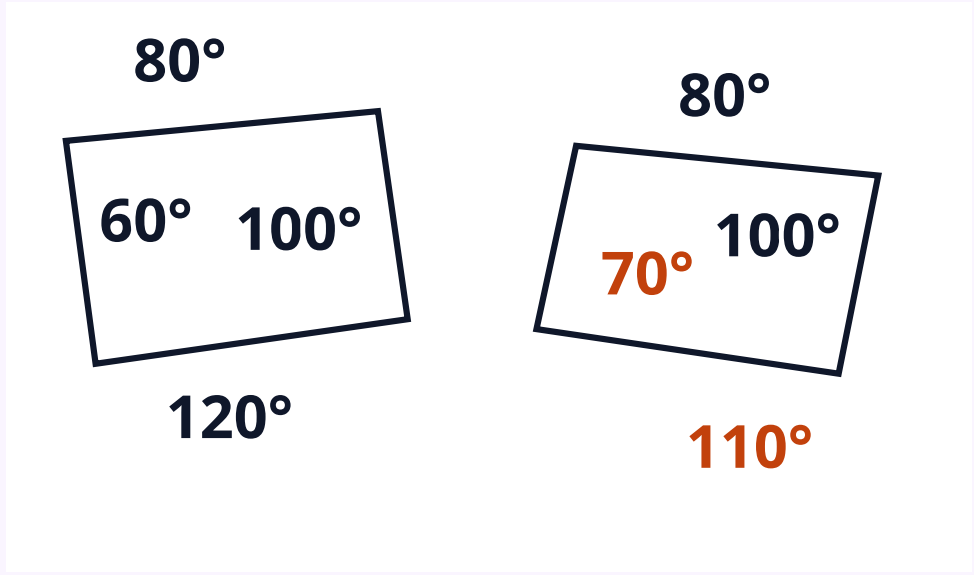
A window on the same plan

- A window measures 2.0 centimetres on the 1 to 50 plan.
- Multiply 2.0 by 50 to get 100 centimetres on the ground.
- That 100 centimetres is 1.0 metre, so the real window is 1.0 metre wide.



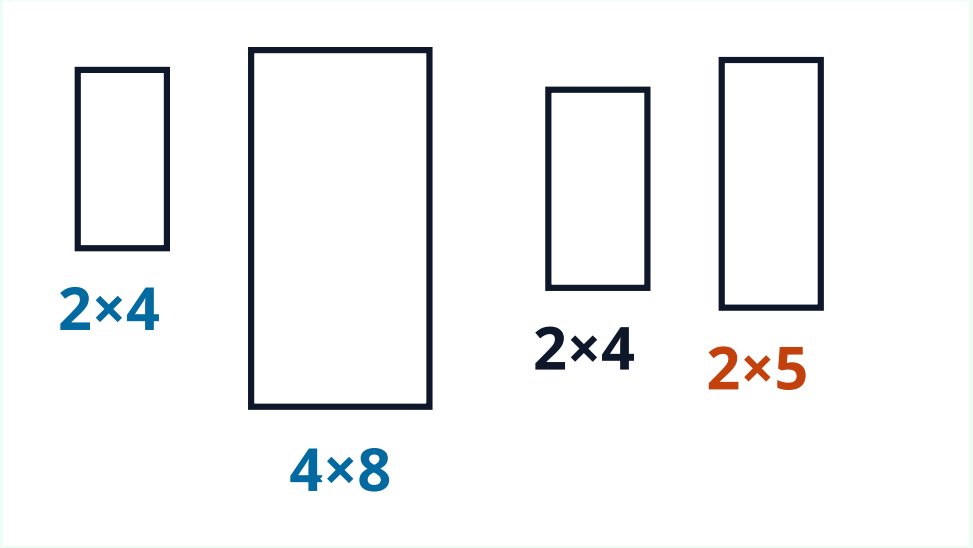
A quadrilateral angle check

- One quadrilateral has angles 80, 100, 120, and 60 degrees.
- A second has angles 80, 100, 110, and 70 degrees.
- The 120 degree angle has no match in the second polygon, so the angle test already fails.



Similarity check

- Explain why the 2 by 4 rectangle and the 4 by 8 rectangle pass the side-ratio test.
- Explain why the 2 by 5 rectangle fails against the 2 by 4.
- Find the plan length of an 8.0 metre wall at a scale of 1 to 50.



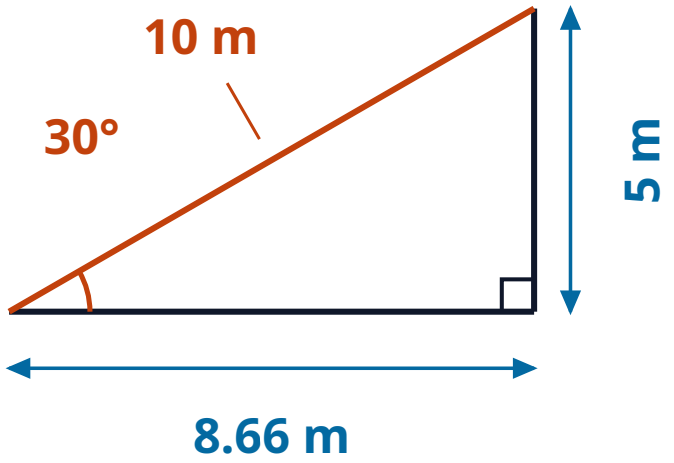
Mathematics 10-3

Unit G: Geometry and trigonometry

Primary trigonometric ratios

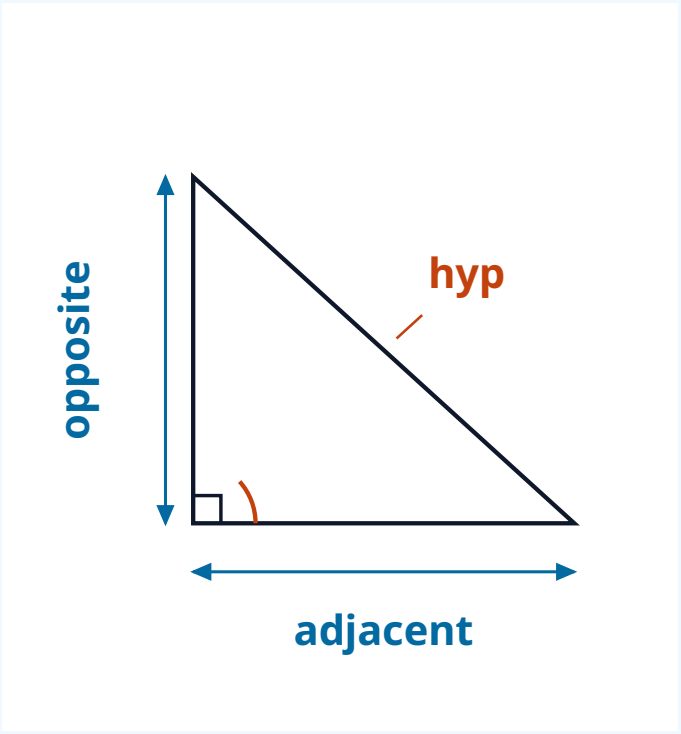
A ladder at 30 degrees

- A 10 metre ladder leans against a wall at 30 degrees to the ground.
- The height it reaches is 5 metres.
- Sine of 30 degrees is 0.5, so height equals 10 times 0.5.
- The same angle on a longer ladder keeps that 0.5 ratio.



Three ratios from similar right triangles

- Opposite means the leg across from the angle.
- Adjacent means the leg beside the angle.
- Hypotenuse means the longest side.
- Tangent equals opposite divided by adjacent.
- Sine equals opposite divided by hypotenuse.
- Cosine equals adjacent divided by hypotenuse.
- For 30 degrees, sine is 0.5, cosine is 0.866, and tangent is 0.577.
- For 45 degrees, sine and cosine are both 0.707 and tangent is 1.



Key terms

opposite

The leg across from the acute angle you are using. (opp)

adjacent

The leg that sits beside the acute angle and forms that angle with the hypotenuse. (adj)

sine

The ratio of the opposite leg to the hypotenuse for a fixed acute angle. (sin = opp/hyp)

cosine

The ratio of the adjacent leg to the hypotenuse for a fixed acute angle. (cos = adj/hyp)

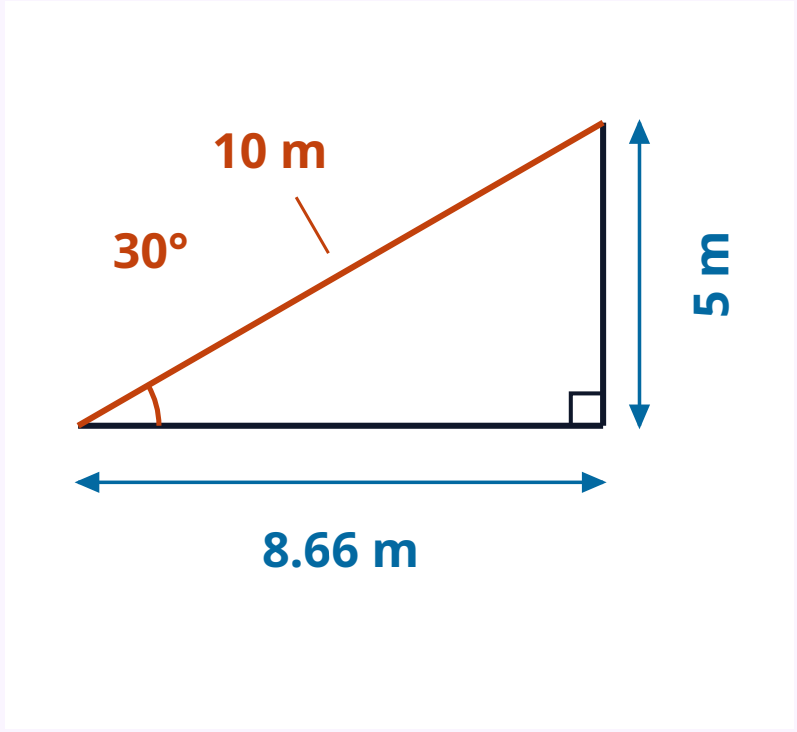
Key terms

tangent

The ratio of the opposite leg to the adjacent leg for a fixed acute angle. (tan = opp/adj)

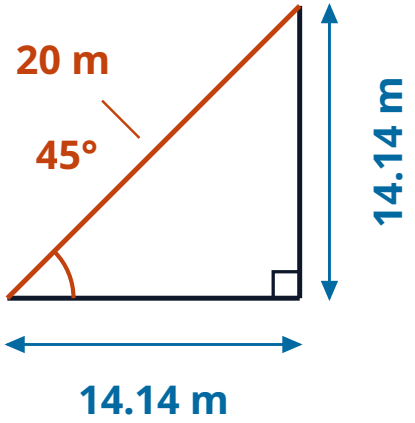
Height and base of the 10 metre ladder

- The height is opposite the 30 degree ground angle, so height equals hypotenuse times sine.
- Height equals 10 times 0.5, which is 5 metres.
- The base is adjacent, so base equals 10 times cosine of 30 degrees.
- That is 10 times 0.866, which is 8.66 metres.



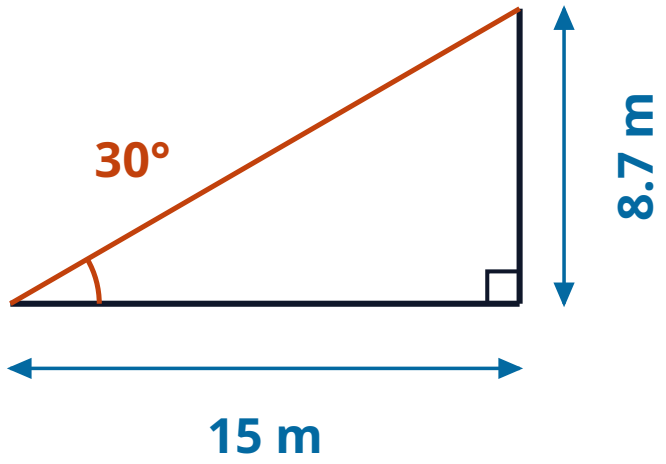
A 20 metre ladder at 45 degrees

- At 45 degrees, sine and cosine are both 0.707, and the two legs match.
- Height equals 20 times 0.707, which is 14.14 metres.
- The base equals 20 times 0.707 as well, so the base is also 14.14 metres.



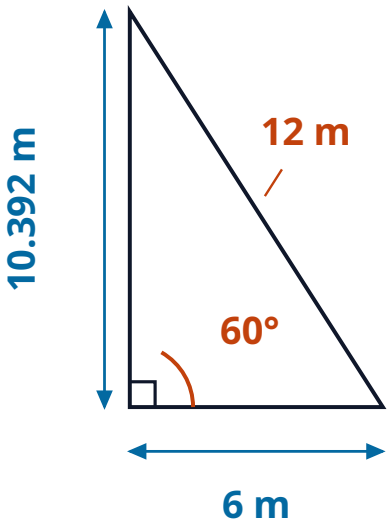
A pole from the ground

- You stand 15 metres from a pole.
- The angle of elevation to the top is 30 degrees.
- The 15 metre ground distance is the adjacent side.
- The pole height is the opposite side.
- Height equals 15 times tangent of 30 degrees.
- Tangent of 30 is 0.577, so 15 times 0.577 equals 8.655 metres, about 8.7 metres.



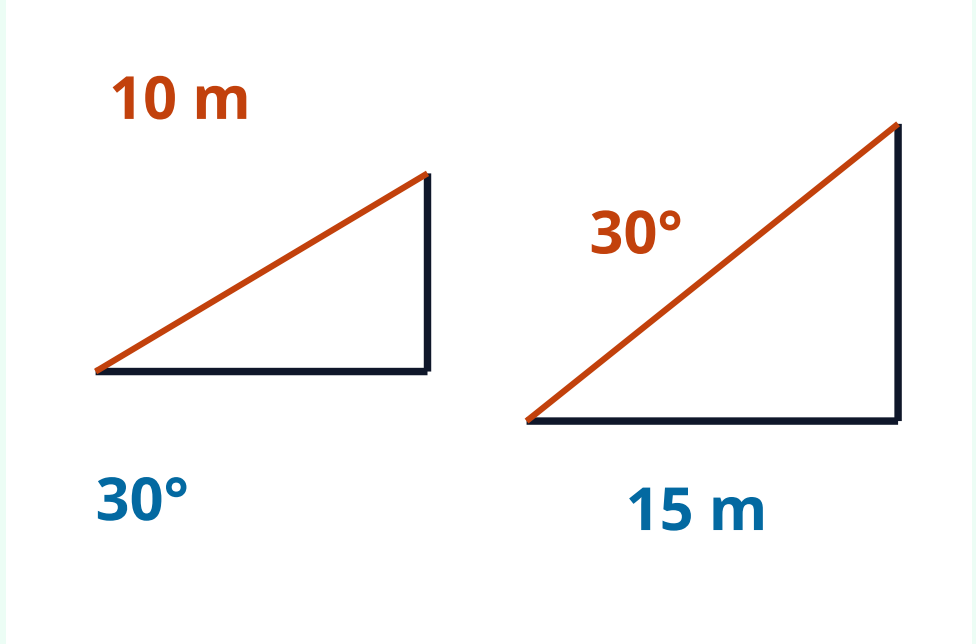
A 12 metre hypotenuse at 60 degrees

- Sine of 60 degrees is 0.866 and cosine of 60 degrees is 0.5.
- The opposite side equals 12 times 0.866, which is 10.392 metres.
- The adjacent side equals 12 times 0.5, which is 6 metres.
- A sensible check is that both legs are shorter than the 12 metre hypotenuse.



Ratio check

- Find the wall height for a 10 metre ladder at 30 degrees to the ground.
- Find the pole height when you stand 15 metres away at an elevation of 30 degrees.
- Use tangent 0.577.



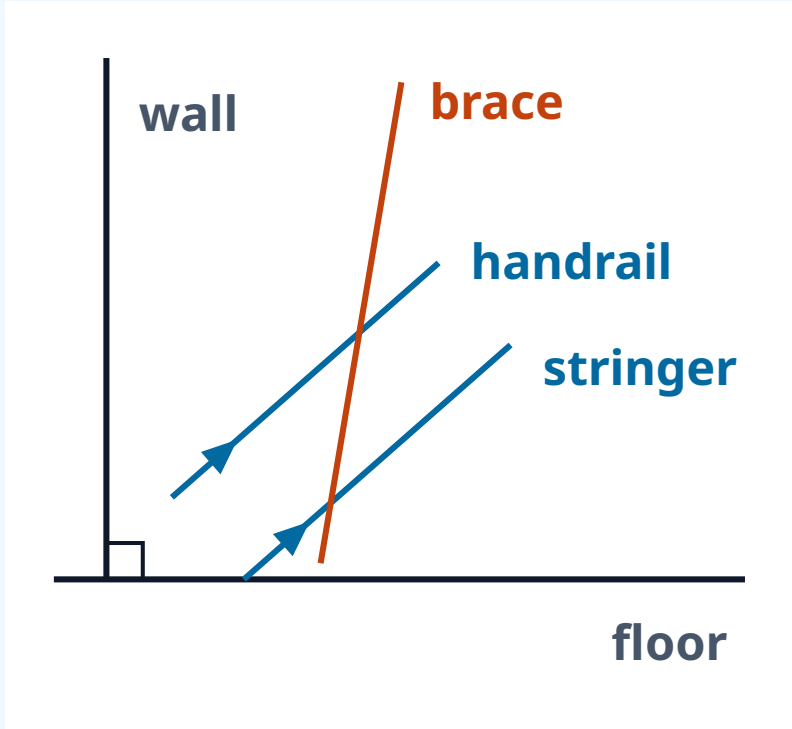
Mathematics 10-3

Unit G: Geometry and trigonometry

Parallel lines and transversals

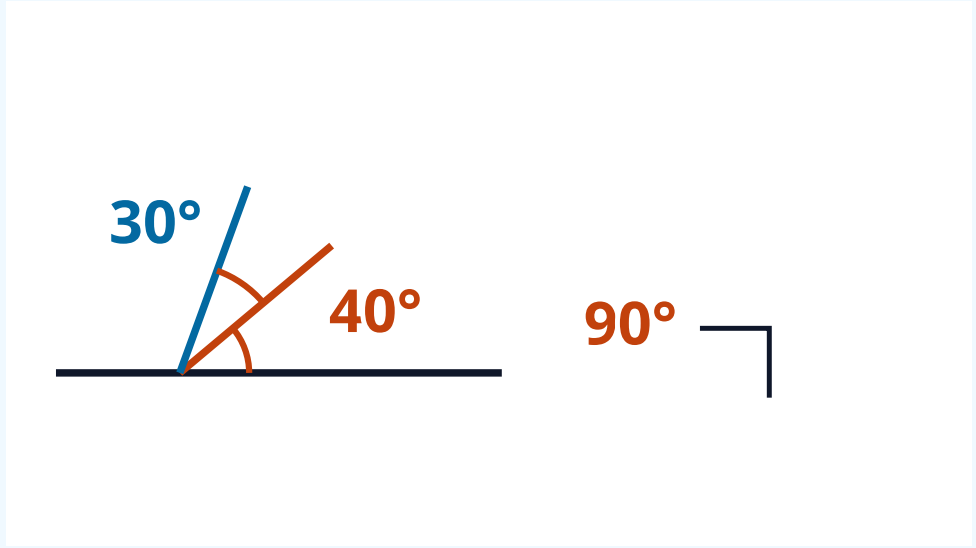
Parallel lines in a stair

- On a stair, the handrail runs the same way as the stringer, so those two lines are parallel.
- The wall meets the floor in a square corner, so the wall is perpendicular to the floor.
- A diagonal brace cuts across both lines.
- The brace is a transversal, a line that crosses two or more other lines.



Angle pairs on a crossing line

- Complementary angles add to 90 degrees.
- Supplementary angles add to 180 degrees.
- A transversal is the line that cuts across two others and makes a pair of angles at each crossing.
- When the two lines are parallel, corresponding angles match, and same-side interior angles add to 180 degrees.



Key terms

parallel lines

Lines that keep a steady gap, like a handrail and the stringer it follows. (||)

transversal

A line that crosses two or more other lines and creates pairs of angles at the crossings. (t)

corresponding angles

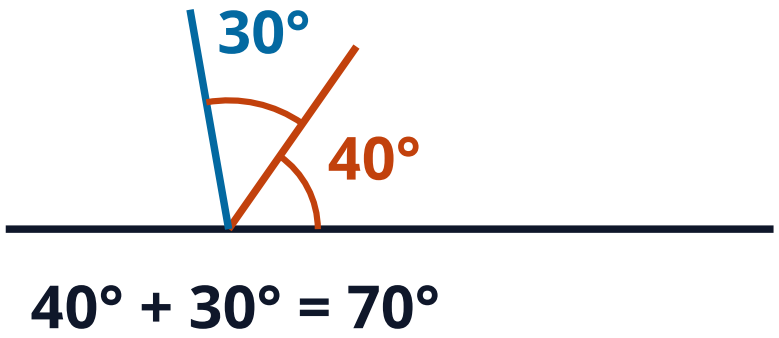
Angles that sit in the same corner at each crossing of a transversal, such as both upper-left angles. (F shape)

same-side interior angles

The two angles between the parallel lines and on the same side of the transversal. They add to 180 degrees. (sum 180)

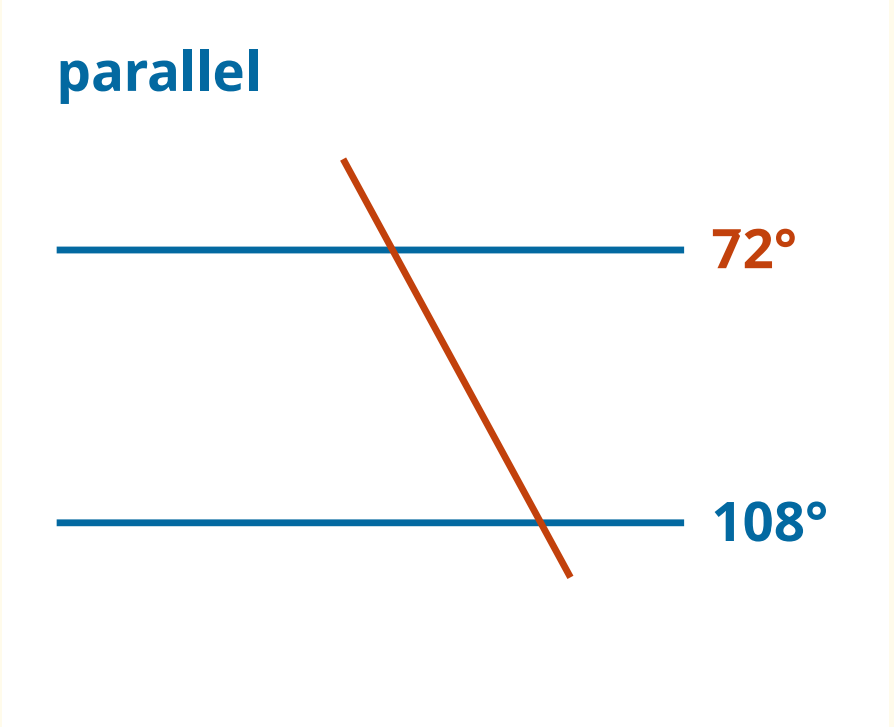
A 40 degree angle beside a 30 degree angle

- Two neighbouring angles measure 40 degrees and 30 degrees.
- Their sum is 70 degrees.
- A complementary pair needs 90 degrees and a supplementary pair needs 180 degrees, so this pair is a plain neighbouring pair.



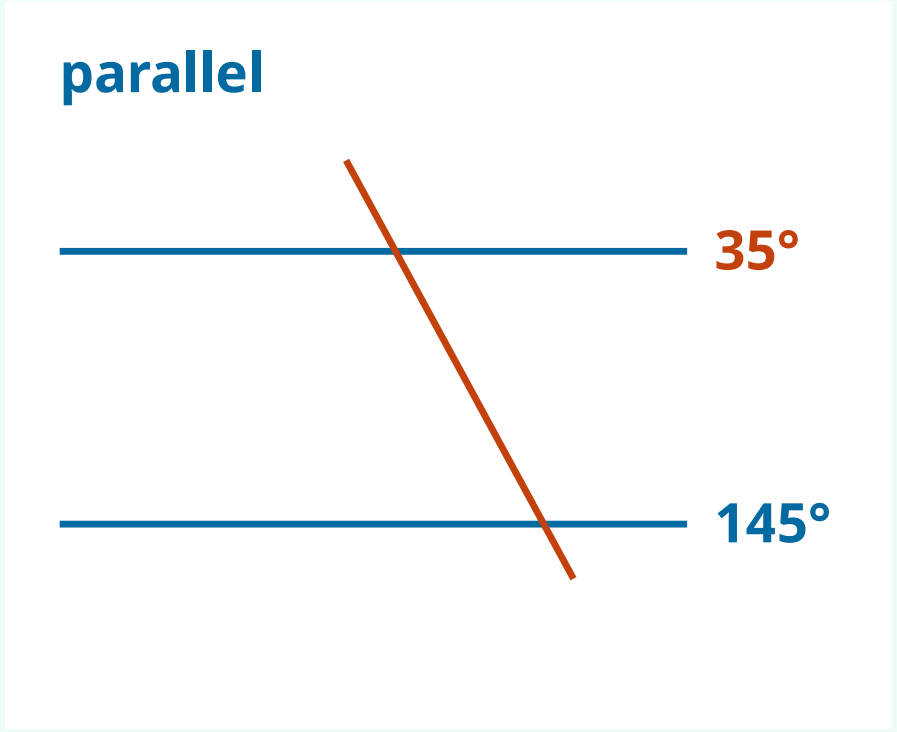
A 72 degree stair cut

- Two parallel rails are cut by a transversal, and one interior angle is 72 degrees.
- The corresponding angle and the alternate interior angle are also 72 degrees.
- The same-side interior angle is 180 minus 72, which is 108 degrees.



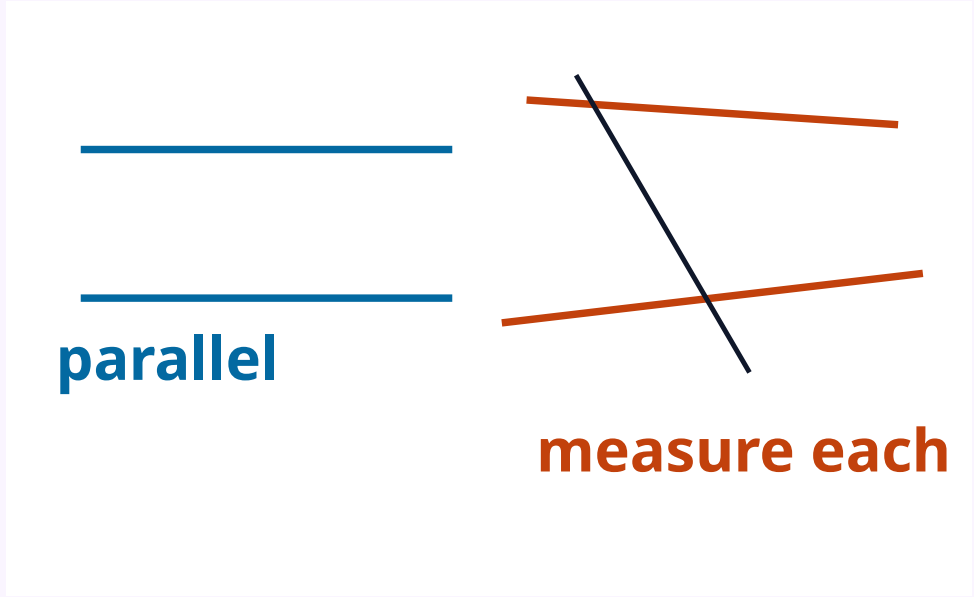
A 35 degree handrail

- A handrail is parallel to the stringer and a brace crosses it at 35 degrees.
- The angle that matches the 35 degree angle is 35 degrees.
- The same-side interior angle is 180 minus 35, which is 145 degrees.



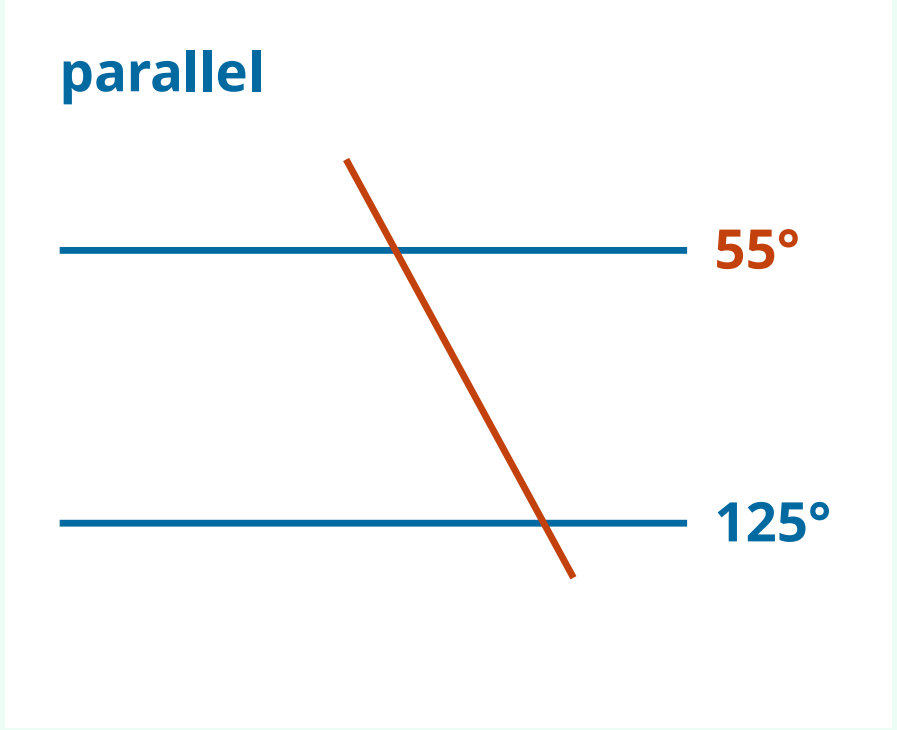
When the rails leave the parallel test

- Equal corresponding angles and 180 degree same-side pairs belong to a parallel pair of lines.
- A tilted rail fails that parallel test, so each angle at the crossing gets its own protractor reading.
- A wall that stands square to the floor is perpendicular.
- The right angle there is 90 degrees.



Angle check

- A stair stringer meets a vertical post at 55 degrees, and the stringer is parallel to the handrail.
- Find the same-side interior angle.
- Sort a wall and a floor, a handrail and a stringer, and a diagonal brace.



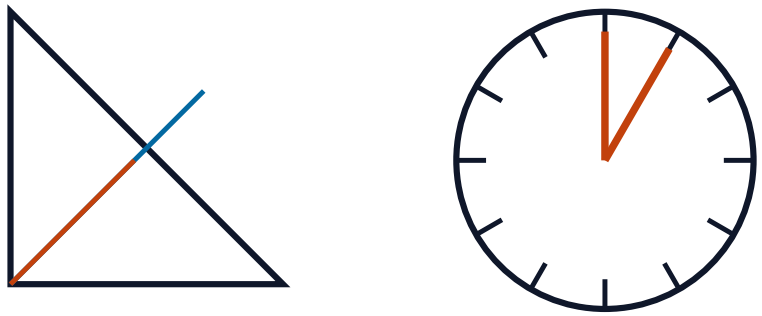
Mathematics 10-3

Unit G: Geometry and trigonometry

Drawing and bisecting angles

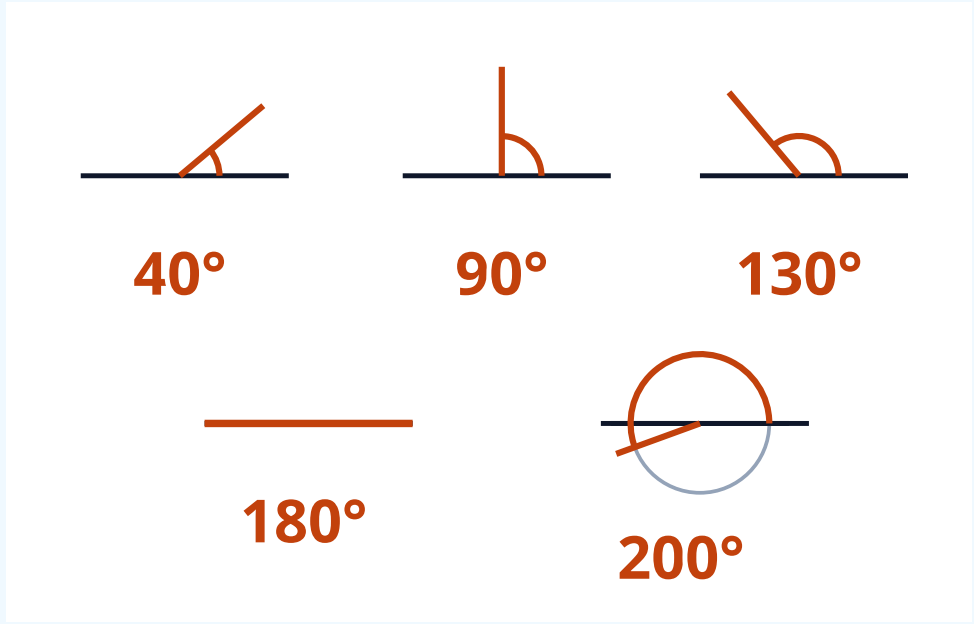
A paper corner and a clock

- A square corner of paper is 90 degrees.
- Fold it in half and the crease is 45 degrees.
- Fold that half again and the crease is 22.5 degrees.
- On a clock, the hour marks are 30 degrees apart, because 360 divided by 12 is 30.



Five angle names and a protractor

- An acute angle is under 90 degrees.
- A right angle is 90 degrees.
- An obtuse angle sits between 90 and 180.
- A straight angle is 180.
- A reflex angle sits between 180 and 360.



Key terms

acute angle

An angle smaller than a square corner, so its measure is under 90 degrees. (< 90)

obtuse angle

An angle larger than a square corner and smaller than a straight line, between 90 and 180 degrees. (90 to 180)

reflex angle

An angle that goes past a straight line and stops before a full turn, between 180 and 360 degrees. (180 to 360)

bisect

To cut an angle into two equal angles, such as a 70 degree angle cut into two 35 degree angles. (half)

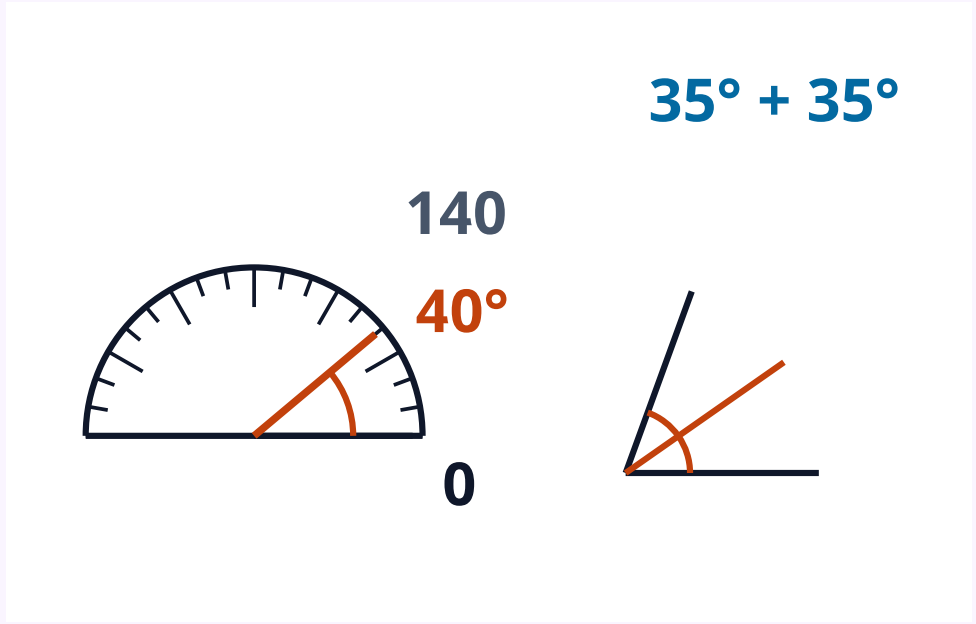
Key terms

protractor

A semicircular tool with two number scales, used by placing the centre on the vertex and reading the scale that starts at 0 on the first ray. (0 to 180)

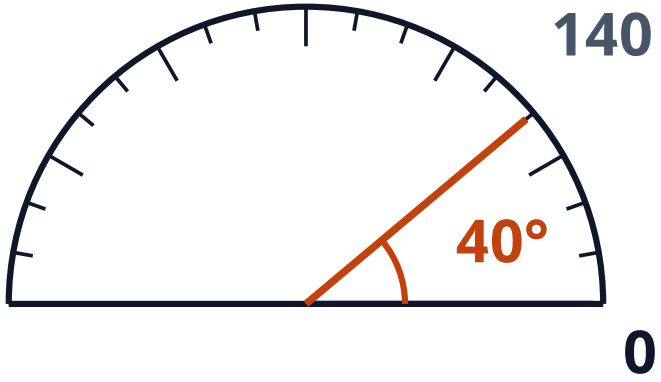
Protractors and angle bisectors

- Read the protractor scale whose 0 sits on the first ray.
- A 40 degree opening is recorded as 40 degrees.
- Bisect means cut into two equal angles.
- A 70 degree angle bisects into two 35 degree angles.



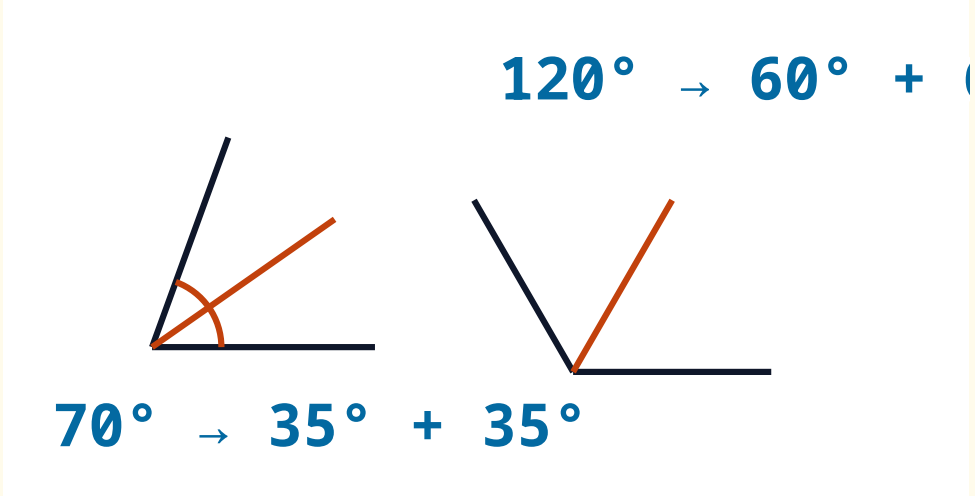
Estimating and measuring 40 degrees

- The referents are 22.5, 30, 45, 60, 90, and 180 degrees.
- A 40 degree opening is a little under the 45 degree paper fold, so the estimate is 40.
- The protractor confirms 40 degrees when the 0 of the scale sits on the first ray.



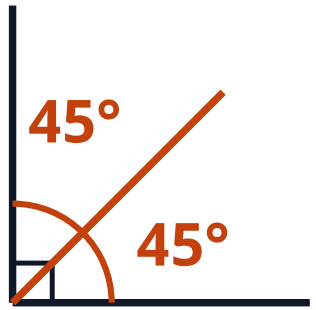
Bisect 70 degrees and 120 degrees

- Half of 70 is 35, so the bisector makes two 35 degree angles.
- Half of 120 is 60, so the bisector makes two 60 degree angles.
- A compass and straightedge, a protractor, or geometry software can each draw that bisector.



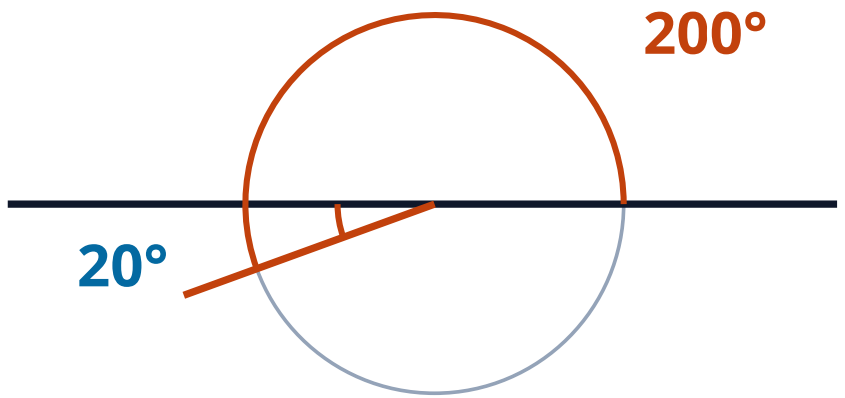
A 90 degree mitre

- A corner trim meets at 90 degrees, and the mitre cuts that corner in half.
- Half of 90 is 45, so each mitre piece is cut at 45 degrees.
- Check the cut against the paper fold.
- The fold and the saw mark should match.



A reflex angle past a straight line

- A straight line is 180 degrees.
- About 20 degrees past that line is about 200 degrees.
- The reflex angle is the long way around, between 180 and 360.
- You can copy an angle with a protractor, a compass and straightedge, a carpenter square, or geometry software.



straight line plus about 20°

Angle check

- Bisect an 80 degree angle and name the two resulting angles.
- A drawn opening looks a little under a 45 degree fold.
- Which protractor reading fits, 40 degrees or 140 degrees?

