Mathematics 10-3 full slide deck

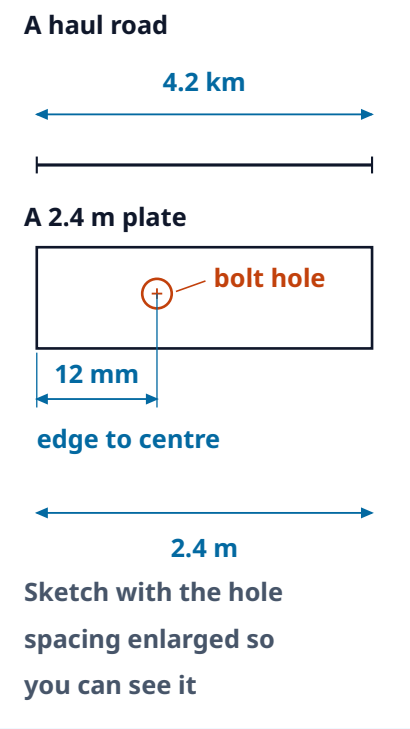
Mathematics 10-3

Unit M: Measurement in SI and imperial

SI prefixes and base units

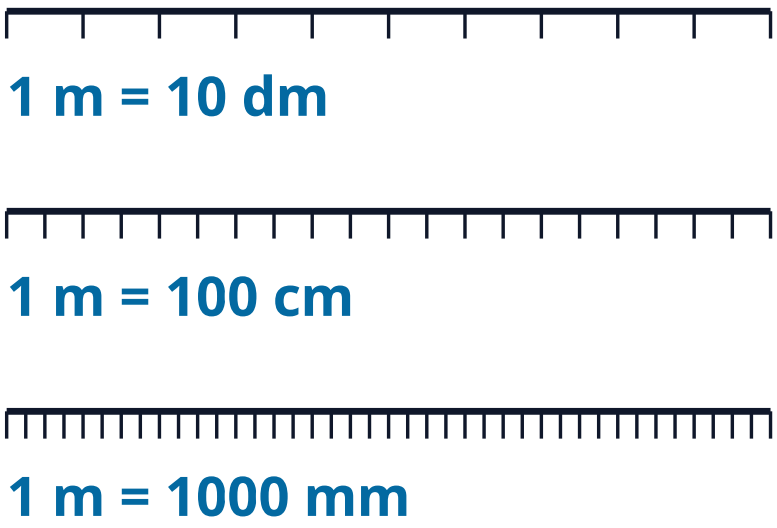
Prefixes in daily use

- A truck log says 4.2 kilometres.
- A shop drawing says the plate is 2.4 metres, with a bolt hole whose centre is 12 millimetres from the edge.
- A clinic label uses milligrams.
- Those jobs all use SI.
- Kilo means 1000.
- Milli means one thousandth.
- The prefix tells you which size of unit you have.



Base units and the six prefixes

- You measure a room in metres, flour in kilograms, and a jug in litres.
- A floor uses square metres.
- A box uses cubic metres.
- Milli means 0.001, centi means 0.01, and deci means 0.1.
- Deca means 10 and its symbol is da.
- Hecto means 100.
- Kilo means 1000.
- The mark 10^{-3} means 10 to the power of −3, which equals 0.001.
- That is why milli lines up with a three-place decimal.



Key terms

SI prefix

A base-ten marker written before a unit to scale it. This course uses milli, centi, deci, deca, hecto, and kilo. (m, c, d, da, h, k)

metre

The SI base unit of length. Other length units in this course are prefixes on the metre. (m)

kilogram

The SI base unit of mass. The name already includes kilo, and a gram is one thousandth of a kilogram. (kg)

litre

The capacity unit on SI job sheets. One litre equals one cubic decimetre. (L)

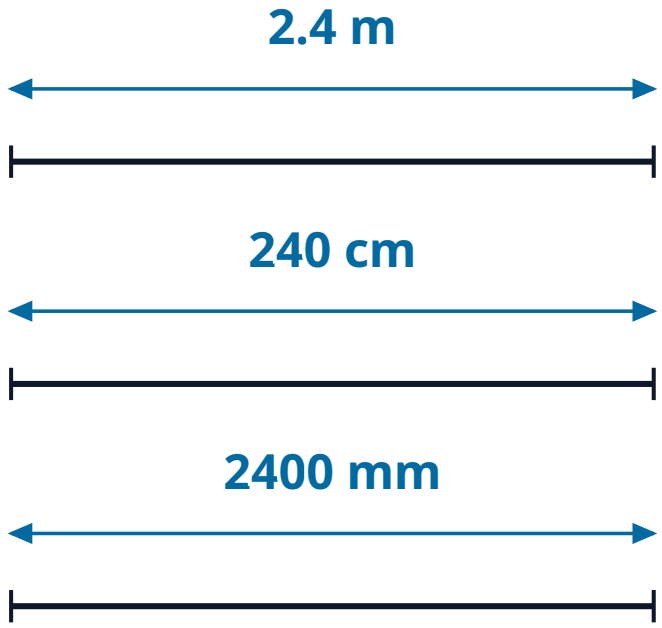
Key terms

kelvin

The SI base unit of temperature. A change of one kelvin equals a change of one degree Celsius. (K)

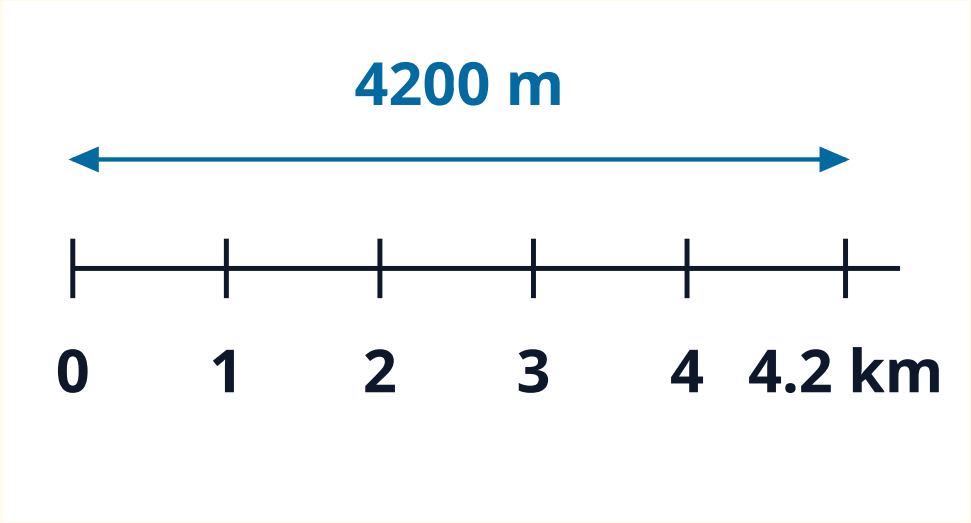
Prefix conversions

- Centimetres are smaller than metres, so multiply by 100.
- Multiplying 2.4 by 100 equals 240 centimetres.
- Millimetres are smaller still, so multiply by 1000.
- Multiplying 2.4 by 1000 equals 2400 millimetres.
- The offset of 12 millimetres, from the plate edge to the bolt-hole centre, divided by 1000 equals 0.012 metres.
- Kilo means 1000, so 4.2 times 1000 equals 4200 metres.



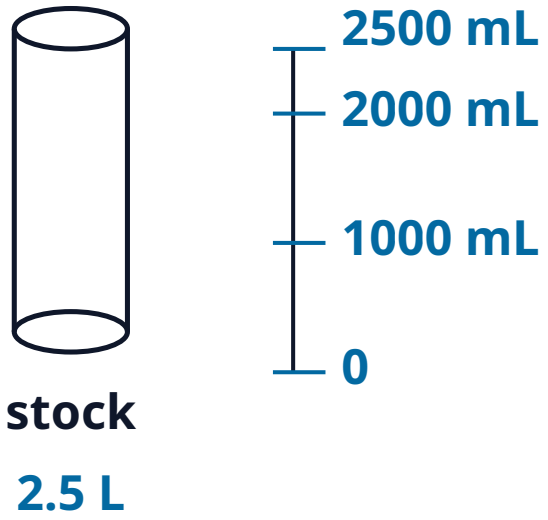
A haul of 4.2 kilometres in metres

- The symbol km stands for kilometre, and the symbol m stands for metre.
- Kilo means 1000, so the haul in metres is 4.2 times 1000.
- Multiplying 4.2 by 1000 equals 4200, so the haul is 4200 metres.



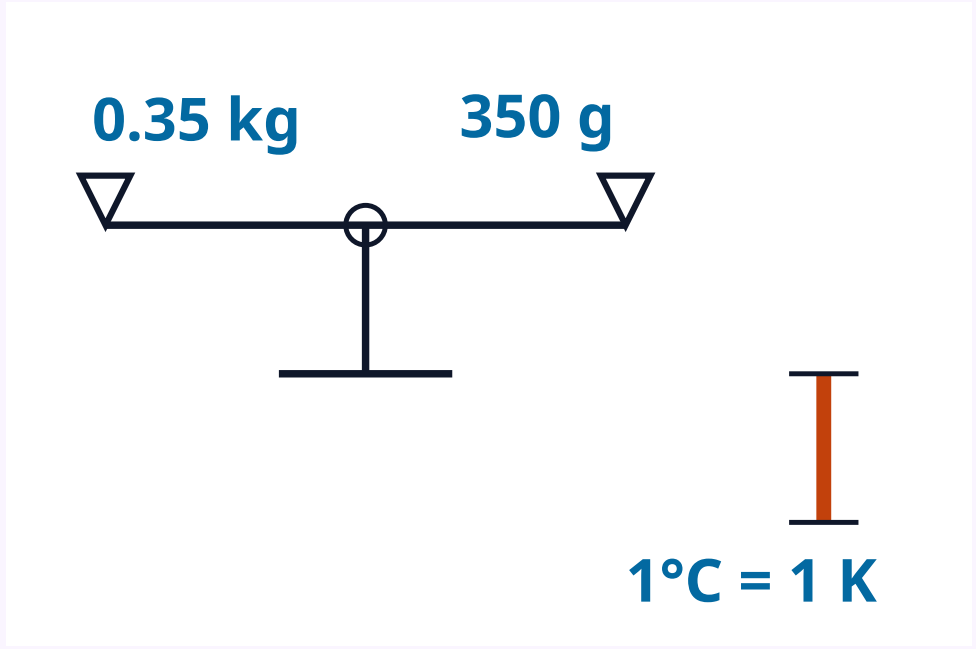
Soup stock in millilitres

- The symbol L stands for litre and the symbol mL stands for millilitre.
- Milli means 0.001, so one litre equals 1000 millilitres.
- Multiplying 2.5 litres by 1000 equals 2500 millilitres of stock.



Mass prefixes and kelvin

- The kilogram already includes the prefix kilo, so a gram is one thousandth of a kilogram and 0.35 kilograms equals 350 grams.
- A 250 milligram tablet equals 0.25 grams because milli means one thousandth and 250 divided by 1000 equals 0.25.
- Job sheets record degrees Celsius, the kelvin is the temperature base unit, and a change of 1 degree Celsius equals a change of 1 kelvin.



Prefix check on the bench

- Rewrite 3.75 metres in centimetres and in millimetres before you leave the ladder.
- State the factor for kilo and use it on a mass or a distance you choose from the lesson.



Mathematics 10-3

Unit M: Measurement in SI and imperial

Imperial units and fractions

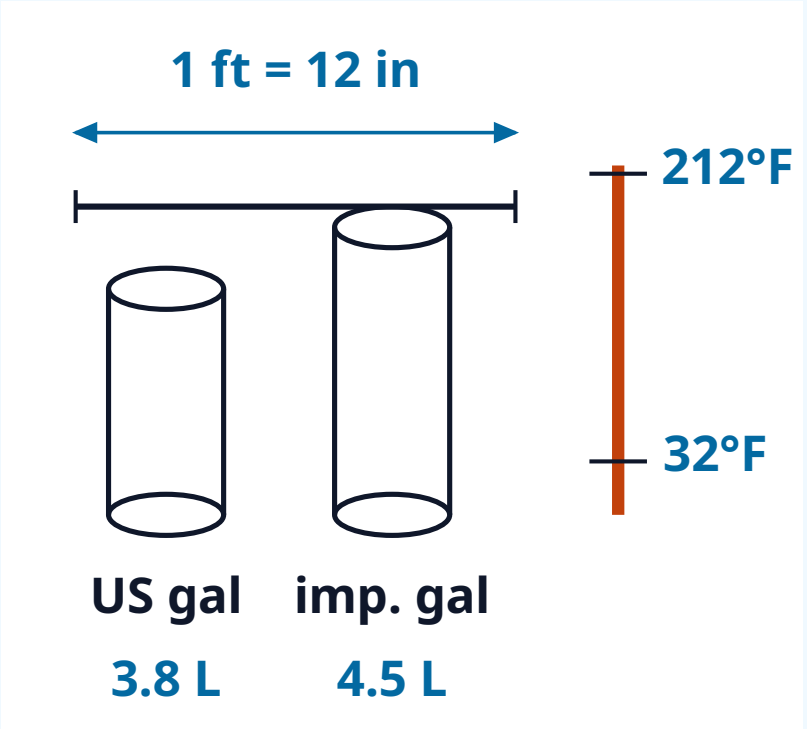
A tape that still speaks inches

- A framer’s tape, a plumbing fitting, and some aviation sheets in Canada still list feet and inches beside SI.
- These units come from older English trade measures.
- A tape splits each inch in half, then in half again: 1/2, 1/4, 1/8, and 1/16.



Imperial units

- One foot equals 12 inches, one yard equals 3 feet and also 36 inches, and one mile equals 1760 yards.
- A floor patch is measured in square feet.
- A box is measured in cubic inches.
- Nails and butter use ounces, and 16 ounces make 1 pound.
- A US gallon is about 3.8 litres, and a British imperial gallon is about 4.5 litres.
- Water freezes at 32 degrees Fahrenheit and boils at 212 degrees Fahrenheit.



Key terms

inch

A small imperial length. Twelve inches make one foot, and the inch is halved into fractions. (in)

foot

An imperial length equal to 12 inches. Three feet make one yard. (ft)

gallon

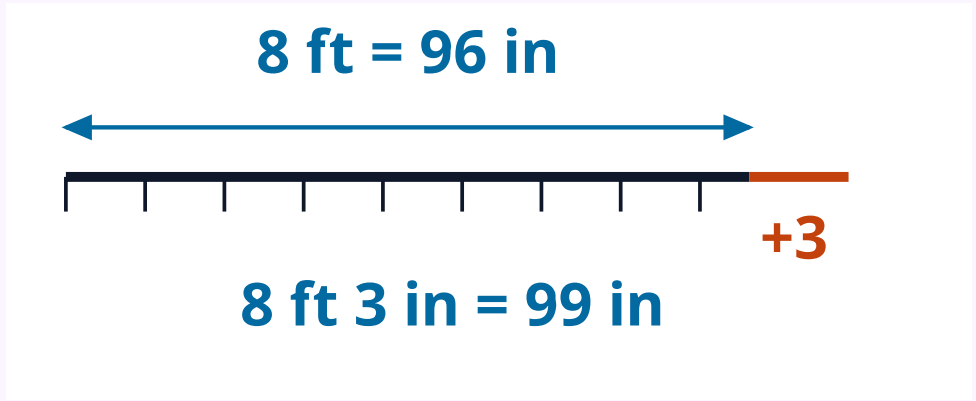
A capacity unit that comes in two trade sizes in this course: the US gallon and the British imperial gallon. (gal)

Fahrenheit

An imperial temperature scale. Water freezes at 32 degrees Fahrenheit and boils at 212. (F)

Feet and inches on the tape

- The symbol ft stands for feet and the symbol in stands for inches.
- Change feet and inches into inches by multiplying the feet by 12 and then adding the extra inches.
- A board of 8 feet 3 inches equals 99 inches, because 8 times 12 equals 96 and 96 plus 3 equals 99.



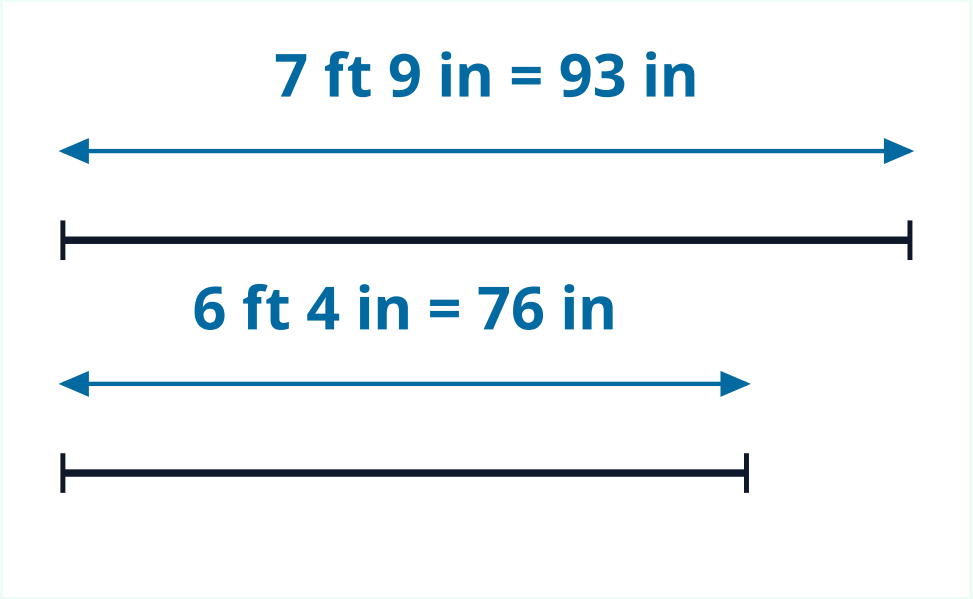
Inches back to feet

- An aviation hose is listed as 54 inches.
- Divide by 12 to find the feet, because one foot equals 12 inches.
- Four groups of 12 use 48 inches, and 54 minus 48 equals 6, so the hose is 4 feet 6 inches.



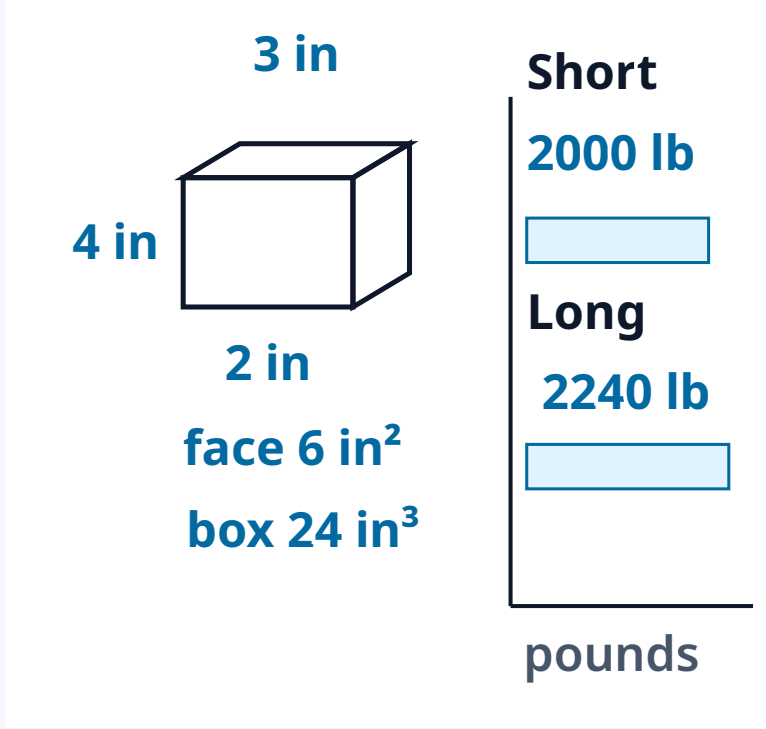
A plumbing run and a doorway

- A plumbing run of 7 feet 9 inches becomes inches when you multiply 7 by 12 and add 9.
- Seven times 12 equals 84, and 84 plus 9 equals 93 inches.
- A doorway height of 6 feet 4 inches equals 76 inches, because 6 times 12 equals 72 and 72 plus 4 equals 76.



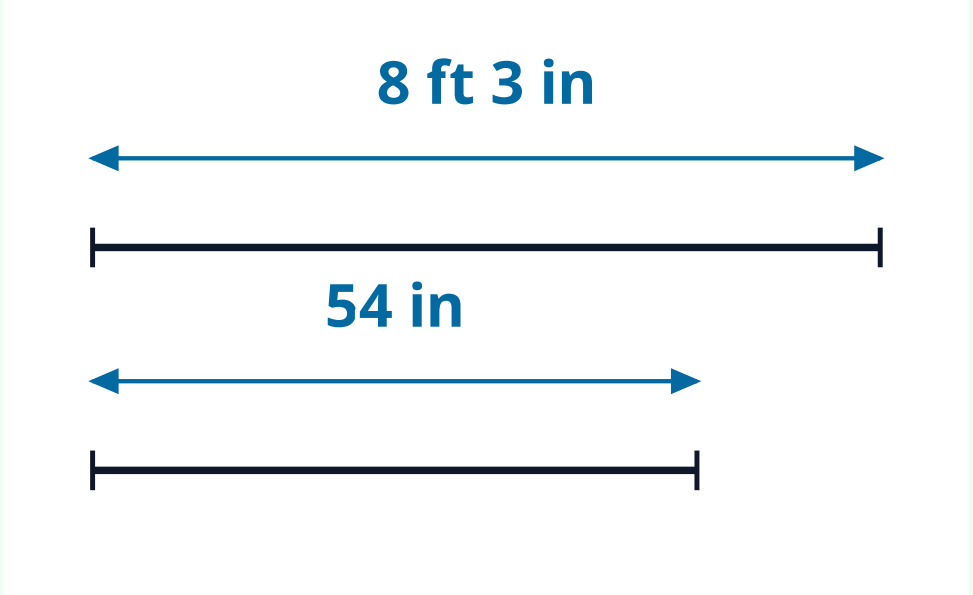
Imperial units of capacity and mass

- A US gallon holds about 3.8 litres and a British imperial gallon holds about 4.5 litres.
- A US short ton equals 2000 pounds and a British long ton equals 2240 pounds.
- A US bushel is about 35 litres and an imperial bushel is about 36 litres.
- One pound equals 16 ounces, so 2 times 16 equals 32 ounces.
- A face of 2 inches by 3 inches has area 2 times 3, which equals 6 square inches.
- A 2 by 3 by 4 box starts with 2 times 3, which equals 6, and 6 times 4 equals 24 cubic inches.



Tape and jug check

- Write 8 feet 3 inches as inches, and write 54 inches as feet and inches.
- State the litre size of a US gallon and of a British imperial gallon.



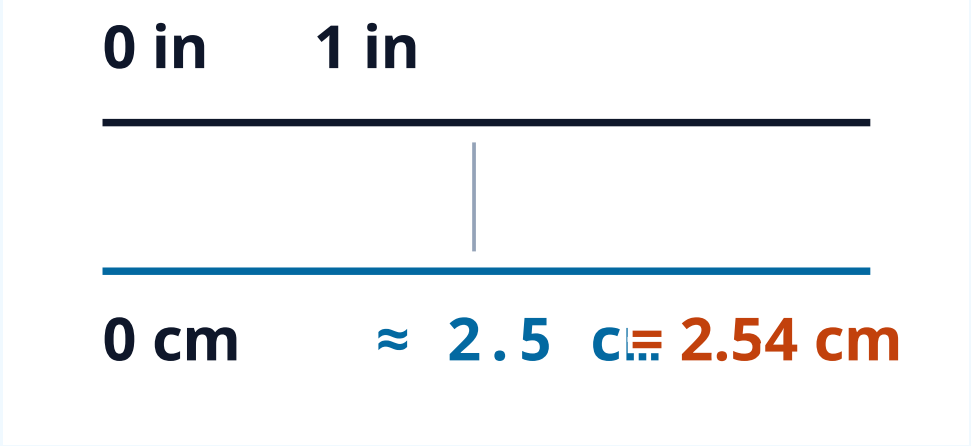
Mathematics 10-3

Unit M: Measurement in SI and imperial

SI and imperial conversion

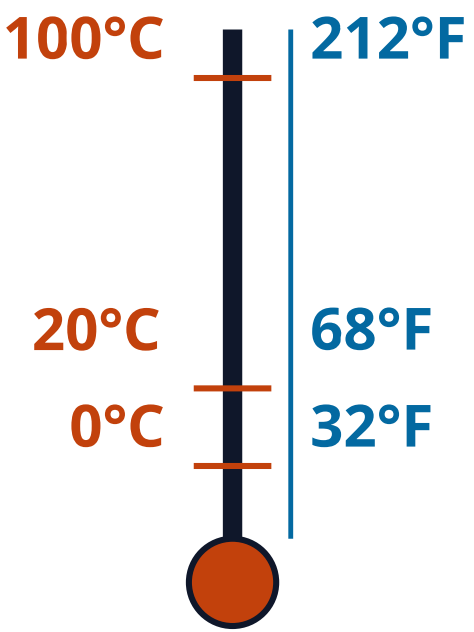
SI and imperial in daily use

- A cabinet drawing labels a stile in inches while the shop rule asks for centimetres, and a prep kitchen log records degrees Celsius.
- Every inch on that drawing uses the same centimetre rate.
- Every Celsius reading on the log uses the same Fahrenheit rule.



Rates that stay proportional

- One inch is approximately 2.5 centimetres as a referent, and a drawing that asks for the trade factor uses 1 inch equals 2.54 centimetres.
- F stands for degrees Fahrenheit and C stands for degrees Celsius.
- The formulas are F = (9/5)C + 32 and C = (5/9) times (F minus 32).
- One litre is approximately one quarter of a US gallon, so 8 litres is about 2 US gallons.



Key terms

centimetre

A hundredth of a metre. Trade drawings in this course use 2.54 centimetres for one inch. (cm)

Celsius

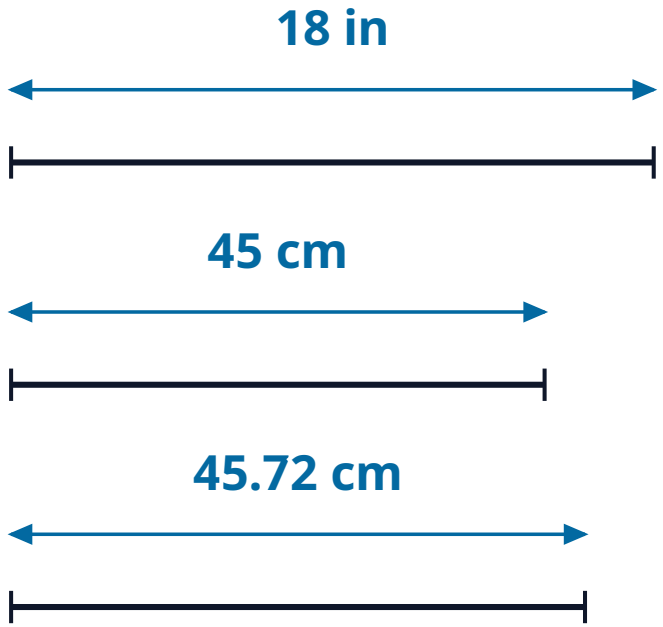
The job-sheet temperature scale. The letter C stands for degrees Celsius. (C)

Fahrenheit

The imperial temperature scale. The letter F stands for degrees Fahrenheit. (F)

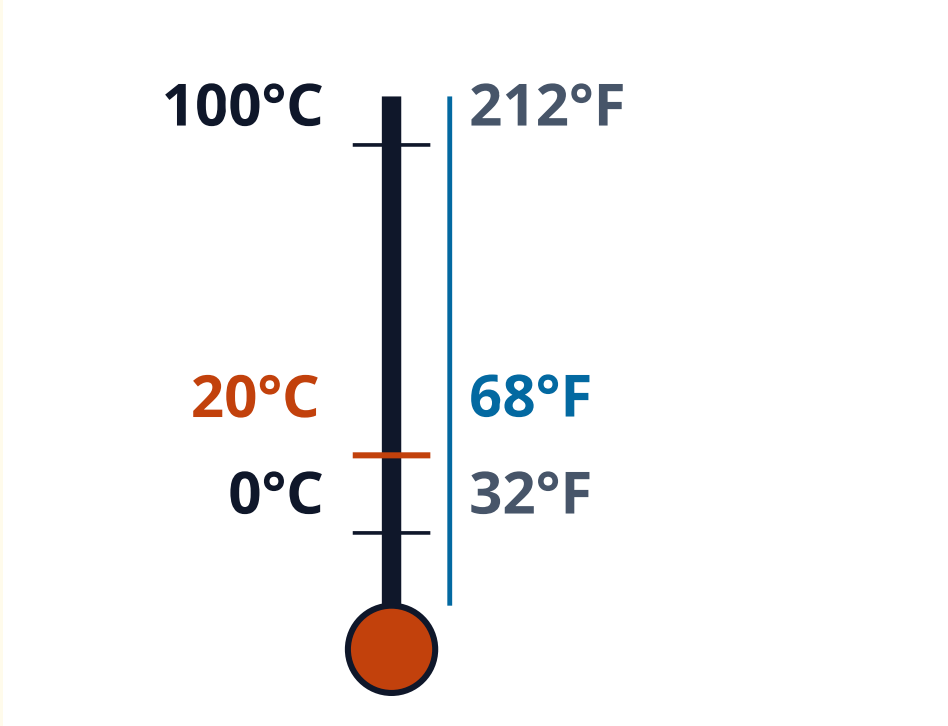
Estimates and exact conversions

- Eighteen inches times 2.5 gives the estimate 45 centimetres.
- Eighteen inches times 2.54 gives the trade length 45.72 centimetres.
- Four inches estimate to 10 centimetres and trade to 10.16 centimetres.
- Twelve inches estimate to 30 centimetres and trade to 30.48 centimetres.



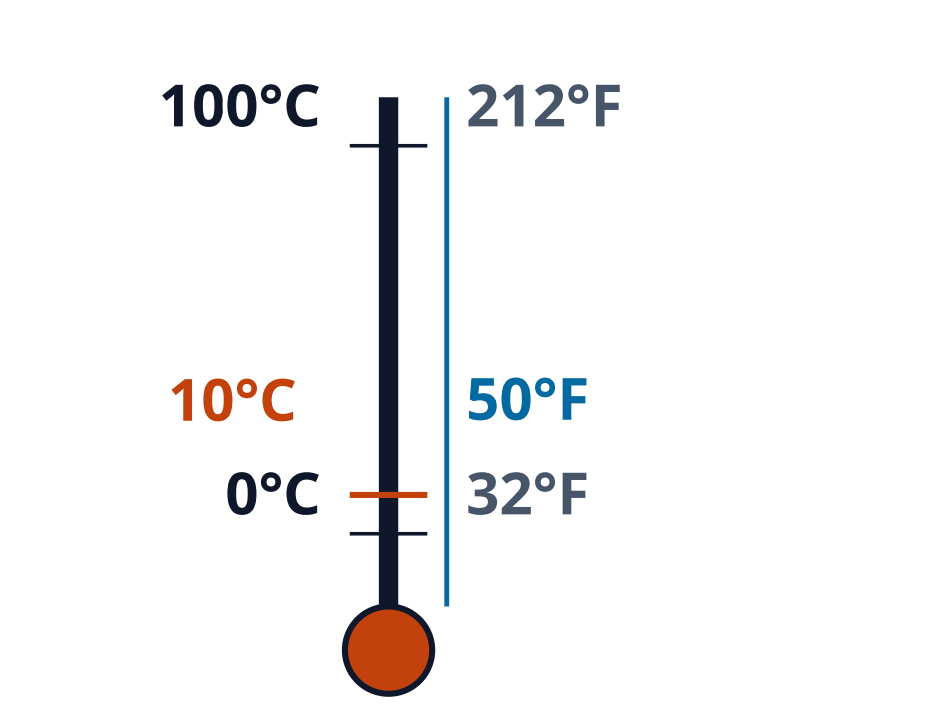
Twenty degrees Celsius to Fahrenheit

- The letter C stands for degrees Celsius and the letter F stands for degrees Fahrenheit.
- Multiply the Celsius reading by 9/5 first.
- That 20 times 9/5 equals 36.
- Then add 32.
- That 36 plus 32 equals 68, so 20 degrees Celsius equals 68 degrees Fahrenheit.



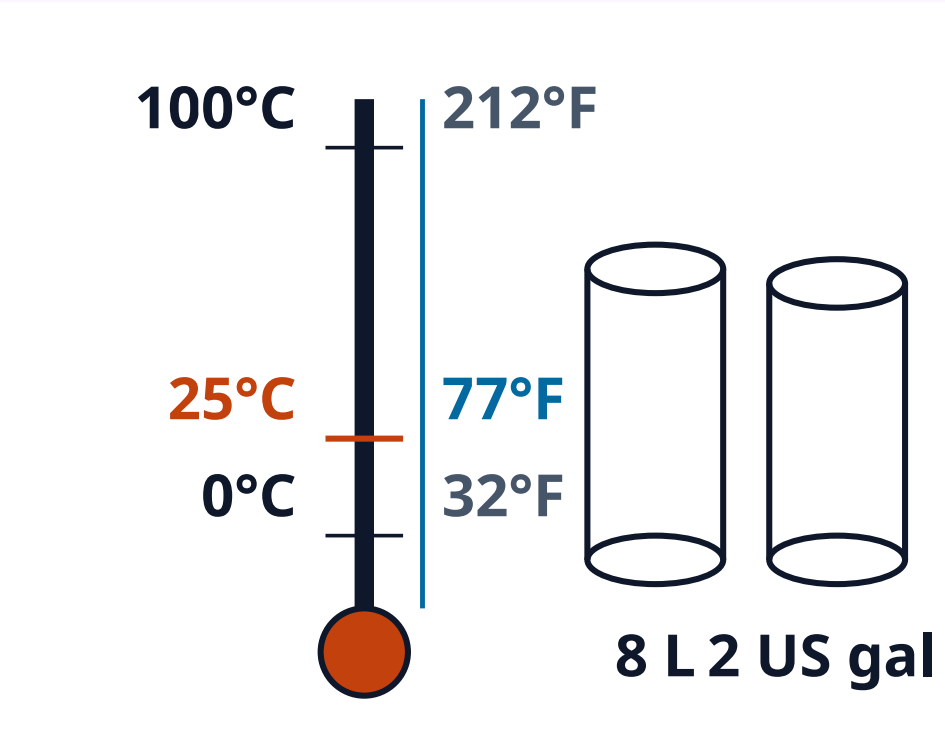
A cooler at 50 degrees Fahrenheit

- The reverse formula is C = (5/9) times (F minus 32).
- Subtract 32 from 50 first.
- That 50 minus 32 equals 18.
- Then multiply by 5/9.
- That 18 times 5/9 equals 10, so 50 degrees Fahrenheit equals 10 degrees Celsius.



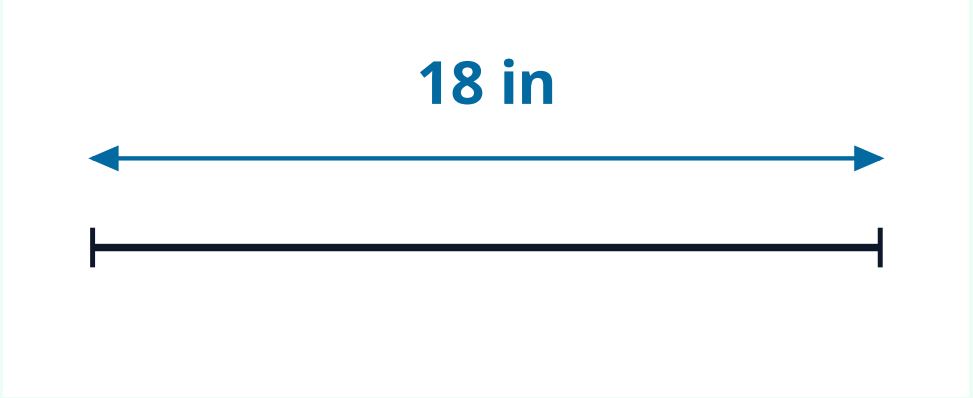
Seventy-seven Fahrenheit and a jug of stock

- A dial at 77 degrees Fahrenheit converts by subtracting 32 from 77 to leave 45, then multiplying 45 by 5/9 to reach 25 degrees Celsius.
- Eight litres is about 2 US gallons because 1 litre is approximately 1/4 US gallon and 8 times 1/4 equals 2.
- The estimate 2.5 centimetres per inch stays a picture.
- The printed factor 2.54 is the machining rate.



Rate check before the cut

- Give both the estimate and the trade centimetre length for 18 inches.
- Convert 20 degrees Celsius to Fahrenheit and show the multiply step before the add.



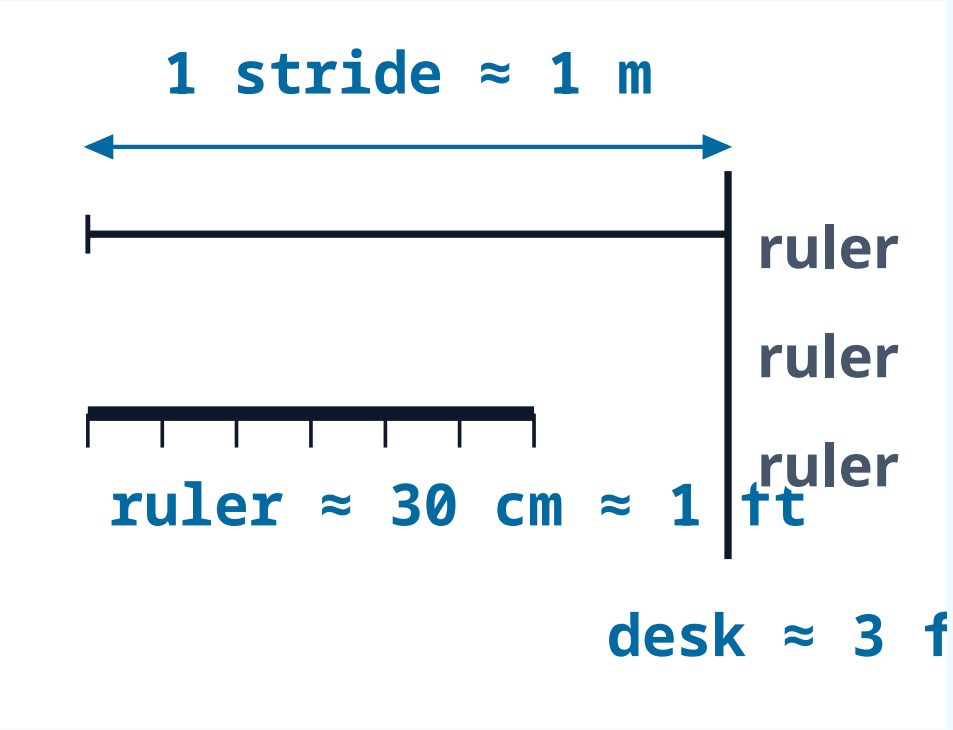
Mathematics 10-3

Unit M: Measurement in SI and imperial

Linear measurement on the job

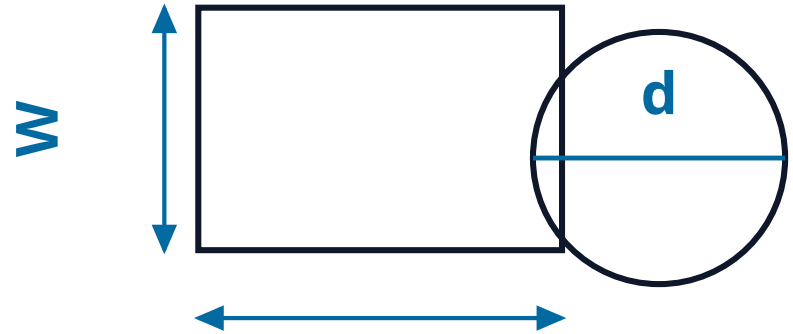
Measuring tools and referents

- A long stride is about 1 metre.
- A ruler is about 30 centimetres, which is also about 1 foot.
- A desk about three rulers tall is about three feet.
- A pipe fitter reads an outside diameter and an inside diameter before ordering a sleeve that has to match the wall.



Referents and perimeter formulas

- OD means outside diameter.
- ID means inside diameter.
- The pipe wall is half of OD minus ID.
- Perimeter walks the outline: P = 2(L + W).
- L is length and W is width.
- Around a circle, C = 3.14 times the diameter.
- C is circumference and 3.14 stands in for pi.
- A shipping size adds length, width, and height.
- The midpoint is half the board.
- Put the measured number in place of each letter.



Key terms

outside diameter

The distance across a pipe or tube measured on the outer surface. (OD)

inside diameter

The distance across the opening inside a pipe or tube. (ID)

perimeter

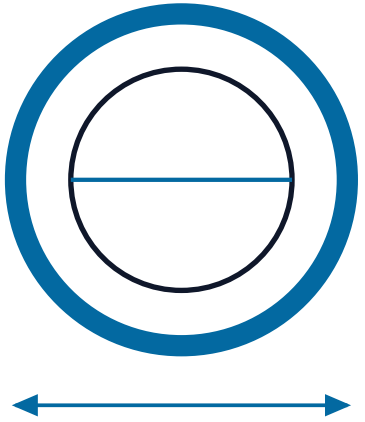
The distance around a flat shape. For a rectangle, P = 2(L + W). (P)

circumference

The distance around a circle. In this course C = pi d with pi taken as 3.14. (C)

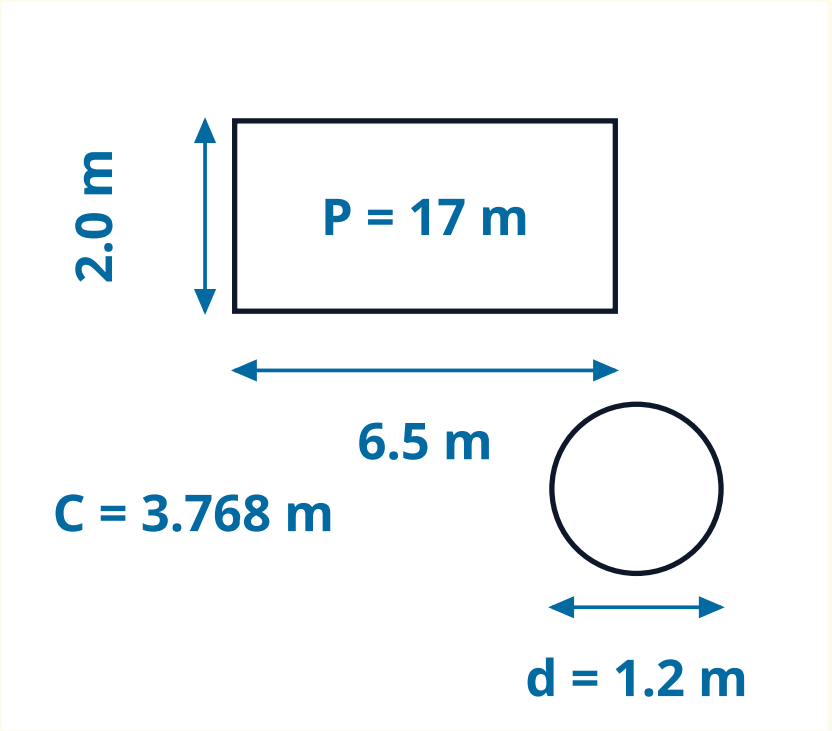
Half the diameter gap

- Outside 5.2 centimetres minus inside 4.6 centimetres equals a gap of 0.6 centimetres.
- Half of 0.6 is 0.3, so the wall thickness is 0.3 centimetres.
- Outside 26 millimetres minus inside 20 millimetres equals a gap of 6 millimetres.
- Half of 6 millimetres is a 3 millimetre wall.



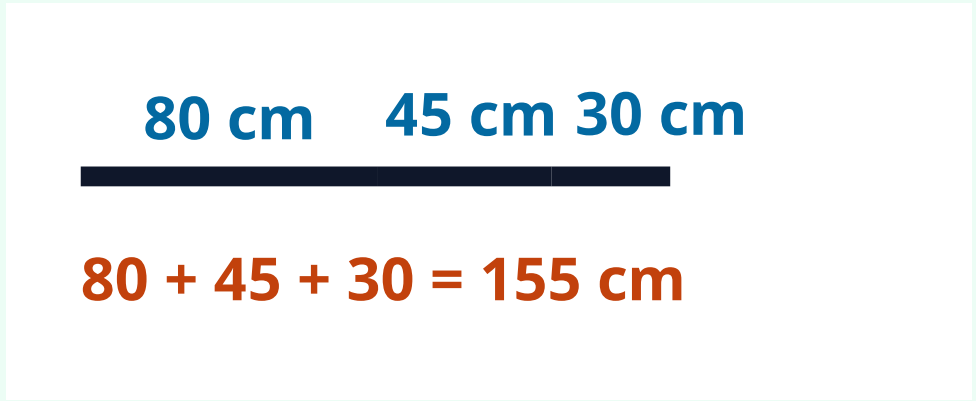
A mat outline and a round lid

- A mat 6.5 metres by 2.0 metres has P = 2(L + W).
- Add 6.5 plus 2.0 to get 8.5, then times 2 to get 17 metres.
- The letter C stands for circumference, d for diameter, and pi is 3.14.
- A diameter of 1.2 metres gives C = 3.14 times 1.2.
- Multiplying 3.14 by 1 equals 3.14 and 3.14 times 0.2 equals 0.628, so the circumference is 3.768 metres.
- A diameter of 0.5 metres gives 3.14 times 0.5 equals 1.57 metres.



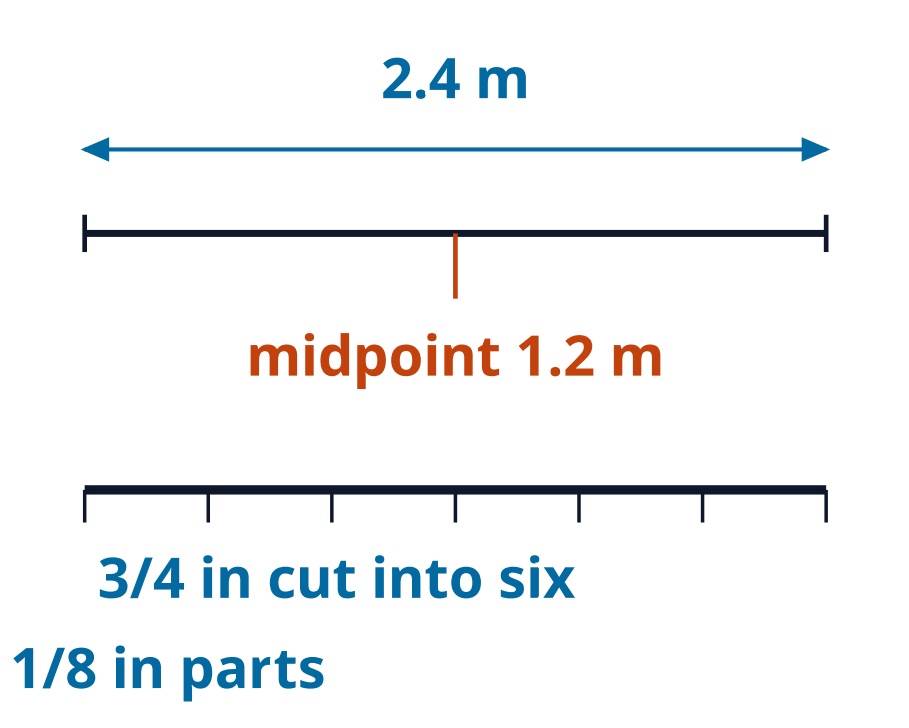
Shipping linear size

- The letters L, W, and H stand for length, width, and height in the shipping sum L + W + H.
- A carton of 80 centimetres, 45 centimetres, and 30 centimetres adds to 155 centimetres.
- A suitcase of 70, 40, and 25 centimetres adds to 135 centimetres, and 135 sits below a stated limit of 158 centimetres.



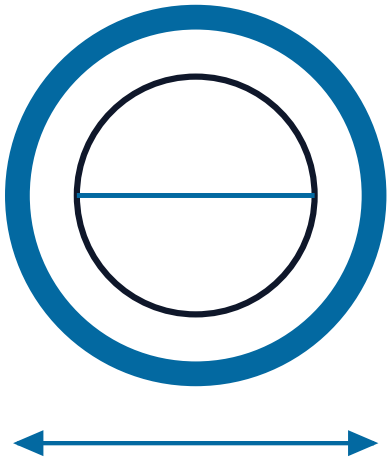
Midpoints and segments

- A midpoint is half the length.
- A 2.4 metre board marks at 1.2 metres, and a 94 inch board marks at 47 inches.
- Dividing 3/4 by 1/8 equals 6, because 3/4 times 8/1 equals 6, so a 3/4 inch strip yields six 1/8 inch segments.
- Adding joins lengths, multiplying counts repeated equal pieces, and dividing by 2 marks a midpoint.
- Compare the result with a referent.



Job-site linear check

- Find the wall for an outside diameter of 26 millimetres and an inside diameter of 20 millimetres.
- Find the perimeter of a 6.5 metre by 2.0 metre mat and say why the result is reasonable.



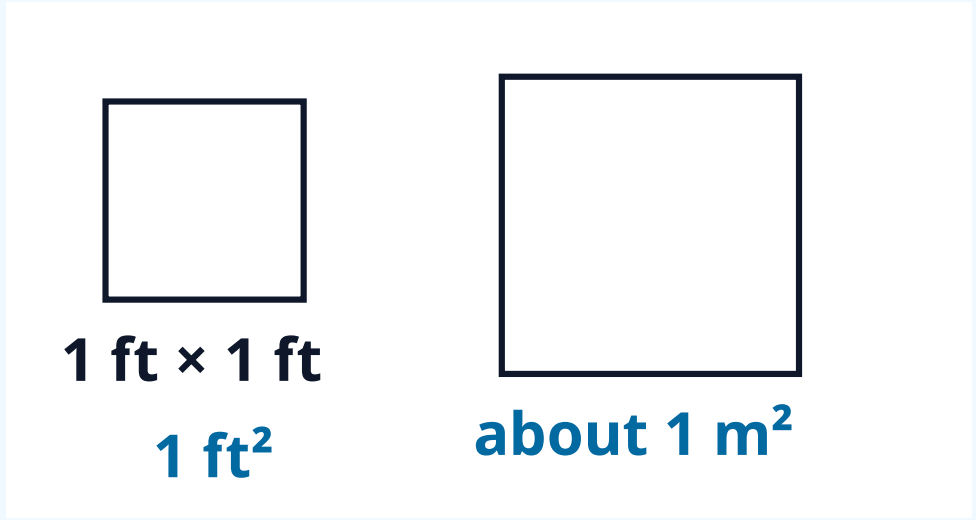
Mathematics 10-3

Unit M: Measurement in SI and imperial

Area of flat shapes

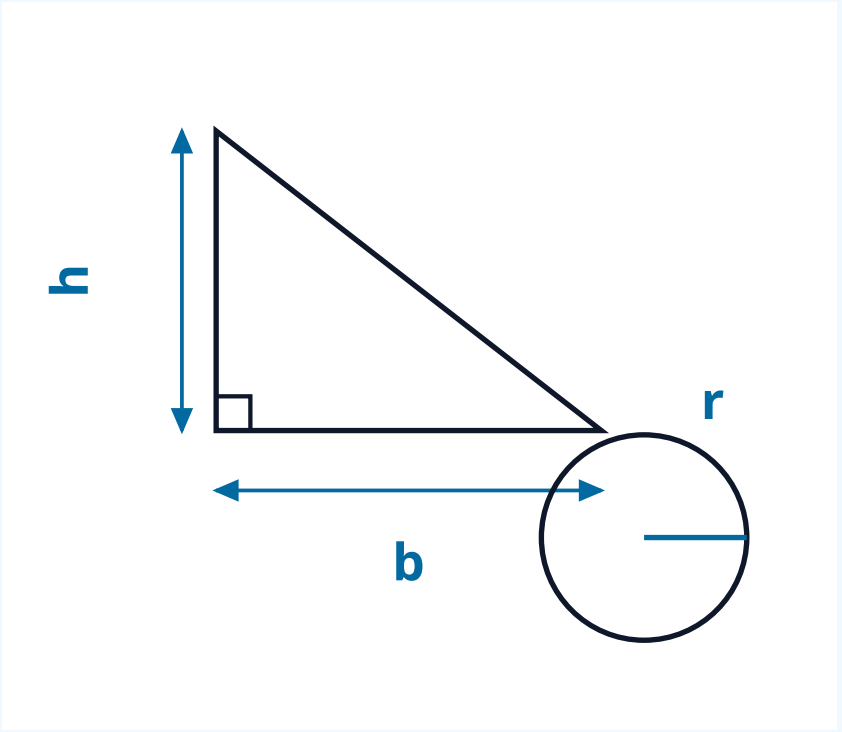
A floor tile and a desk top

- A floor tile 1 foot by 1 foot covers 1 square foot, and a desk top can cover about 1 square metre.
- A painter and a flooring installer both price work from area, so a wild square count wastes material.



Formulas for flat cover

- A rectangle’s area is length times width.
- A means area, l means length, and w means width, so A = l w.
- A triangle’s area is half of base times height.
- The letter b is the base and the letter h is the height.
- For a circle, r is the radius, half the diameter, and A = $3.14r^2$.
- Split an L-shaped room into two rectangles, then add the areas.
- One square metre equals 10 000 square centimetres.



Key terms

square metre

The SI unit of area. A square one metre on each side, written m2. (m2)

radius

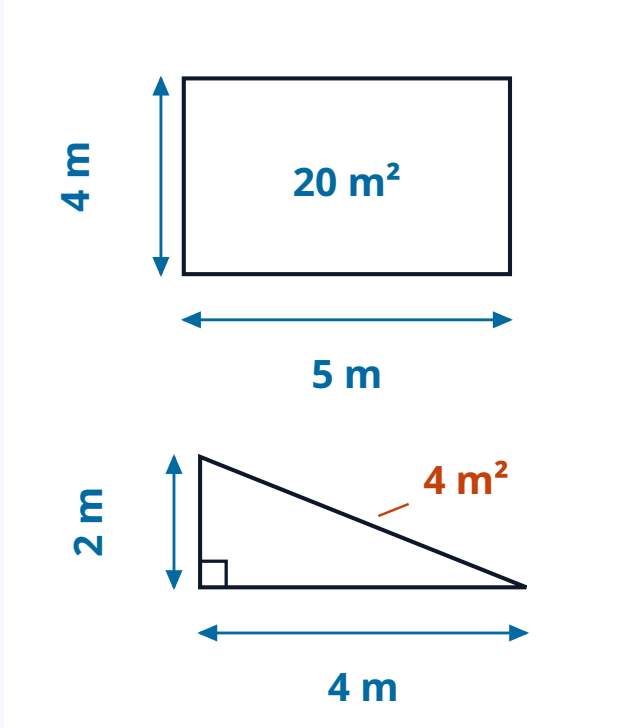
The distance from the centre of a circle to the rim. The radius is half the diameter. (r)

base

The side of a triangle used with the height in the area formula A = (1/2) b h. (b)

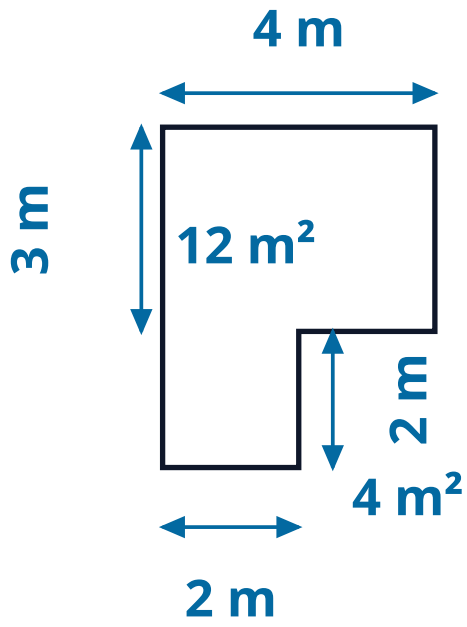
Area of a shed wall

- A square-grid estimate counts full squares and pairs the halves.
- Eight full squares and four half squares estimate 10 square units.
- A rectangle 5 metres by 4 metres has area 5 times 4, which equals 20 square metres.
- A triangle of base 4 metres and height 2 metres has 4 times 2 equals 8, and half of 8 is 4 square metres.
- The shed wall adds those pieces: 20 plus 4 equals 24 square metres.



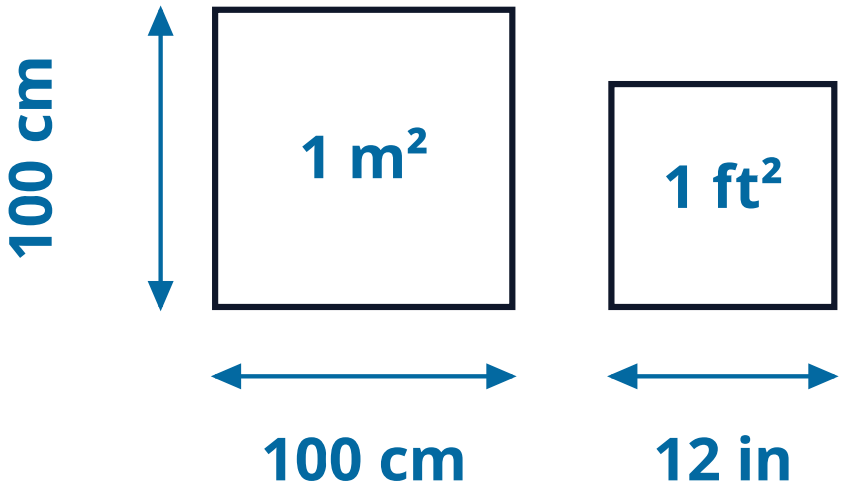
An L-shaped stock room and a bedroom

- Draw the L-shaped room and split it into two rectangles before adding the areas.
- An L-shaped stock room splits into a 4 metre by 3 metre rectangle and a 2 metre by 2 metre rectangle.
- Four times 3 equals 12 square metres.
- Two times 2 equals 4 square metres.
- Twelve plus 4 equals 16 square metres.
- A bedroom 4.8 metres by 3.5 metres splits 3.5 into 3 and 0.5.
- Partial products give 4.8 times 3 equals 14.4, and 4.8 times 0.5 equals 2.4.
- Adding 14.4 and 2.4 equals 16.8 square metres.



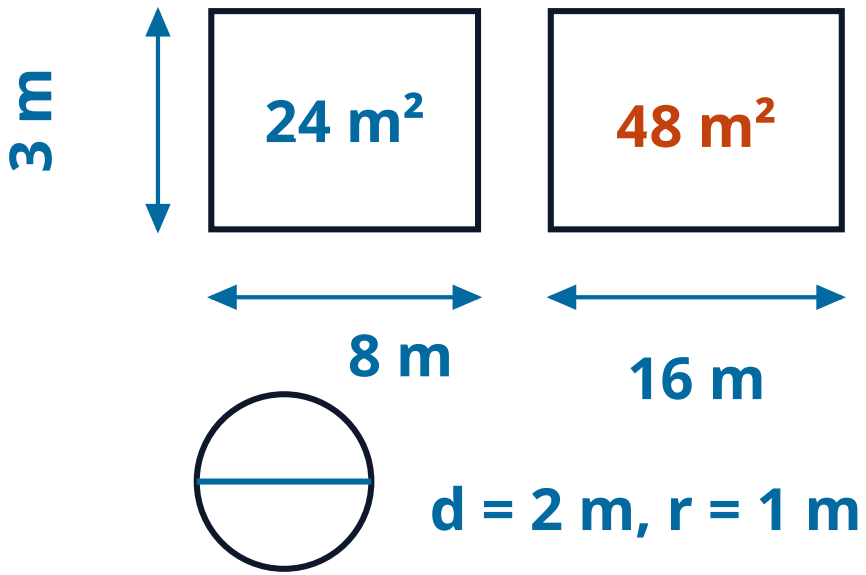
Square units on the order

- The symbol m2 means square metres and cm2 means square centimetres.
- One square metre is a 100 centimetre by 100 centimetre square, so 100 times 100 equals 10 000 square centimetres.
- A patch of 2.5 square metres equals 25 000 square centimetres.
- A patch of 0.4 square metres equals 4000 square centimetres.
- The symbol ft2 means square feet and in2 means square inches.
- One square foot equals 144 square inches, and 3 square feet equal 432 square inches.



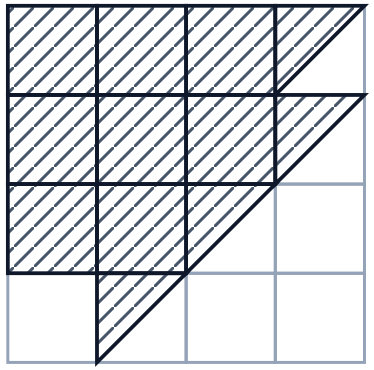
Doubling sides and a round rug

- A rectangle 8 metres by 3 metres has area 24 square metres and perimeter 22 metres, because 2 times (8 plus 3) equals 22.
- Doubling only the length makes 16 by 3.
- Multiplying 16 by 3 equals 48 square metres.
- The new perimeter is 2 times (16 plus 3).
- Sixteen plus 3 equals 19, and 2 times 19 equals 38 metres.
- Doubling both sides makes 16 by 6.
- Multiplying 16 by 6 equals 96, which is 4 times 24.
- A round rug with diameter 2 metres has radius 1 metre.
- A = πr^2 = 3.14 times 1 times 1, which is 3.14 square metres.



Cover check

- Find the triangle area for base 4 metres and height 2 metres, and the shed total of 24 square metres.
- Rewrite 1 square metre in square centimetres and say where the factor 10 000 comes from.



8 full + 4 halves = 10

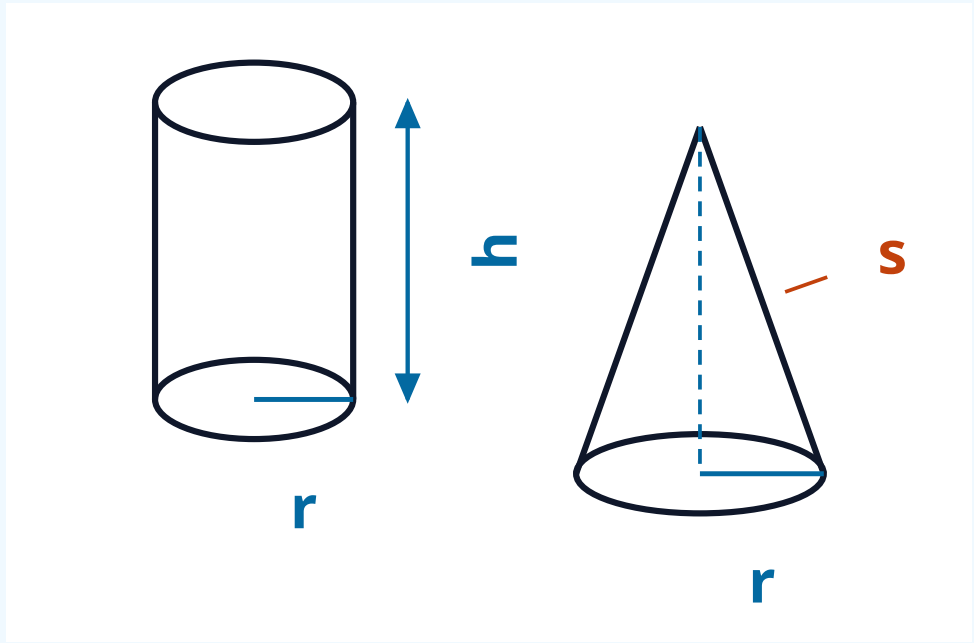
Mathematics 10-3

Unit M: Measurement in SI and imperial

Surface area of solids

Surface area in sheet metal work

- A duct fabricator orders metal for a right cylinder, and a hopper builder orders metal for a right cone.
- You order the metal that covers the outside.
- That cover is the surface area, including the ends when the formula counts the ends.



Cylinder and cone formulas

- Cylinder surface area equals $2\pi r^2$ plus $2\pi rh$.
- The letter r stands for radius, h for height, and pi is taken as 3.14.
- The two circular bases use $2\pi r^2$.
- The lateral face is a rectangle whose width is the circumference $2\pi r$ and whose height is h.
- Cone surface area equals πr^2 plus πrs.
- The letter s stands for slant height, the sloping distance along the side of the cone.



Key terms

surface area

The total cover of the outside faces of a solid, measured in square units. (SA)

slant height

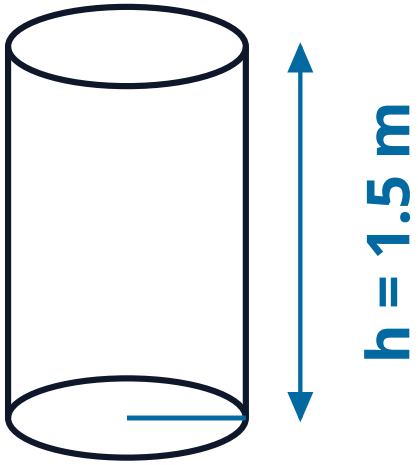
The distance along the sloping side of a right cone, from the base rim to the apex. The letter s stands for it. (s)

radius

Half the diameter. Cylinder and cone formulas in this lesson take the radius, written r. (r)

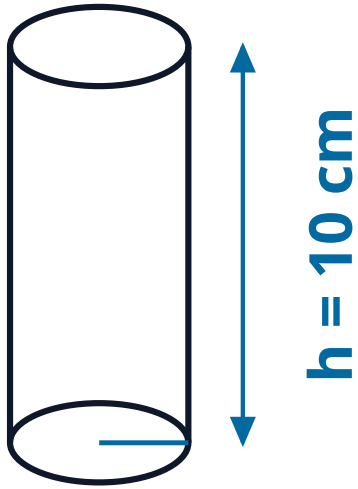
A duct with radius 0.2 metres

- The duct diameter is 0.4 metres, so the radius is half of that, 0.2 metres.
- The height is 1.5 metres and pi is 3.14.
- The two bases are 2 times 3.14 times 0.04, which equals 0.2512 square metres, because 0.2 times 0.2 equals 0.04.
- The lateral area starts with 2 times 3.14 times 0.2, which equals 1.256.
- Then 1.256 times 1.5 equals 1.884 square metres.
- Add 0.2512 and 1.884 to get 2.1352 square metres.



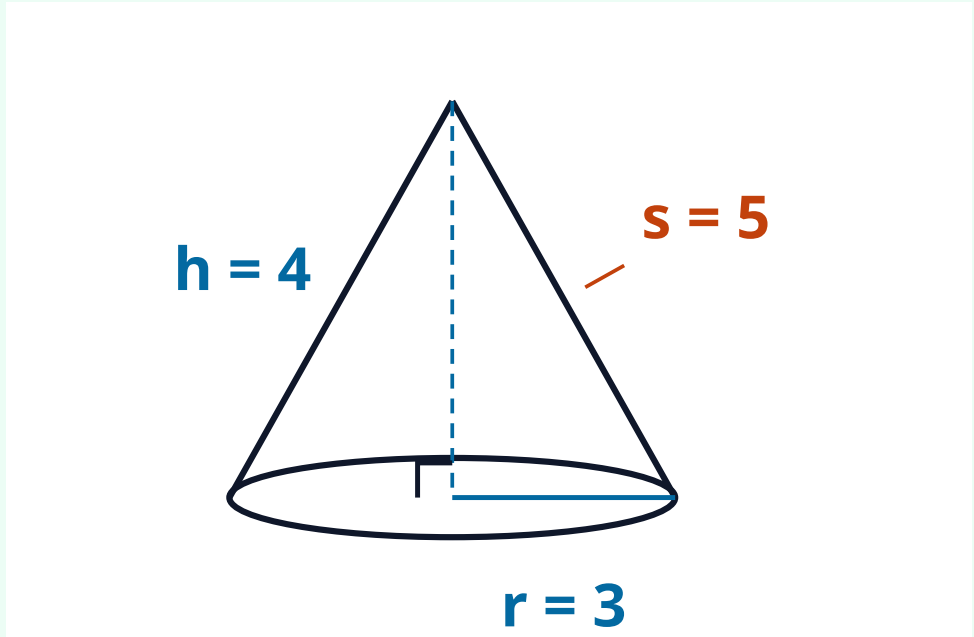
The small cylinder in centimetres

- The small cylinder has radius 2 centimetres and height 10 centimetres.
- The radius squared is 2 times 2, which equals 4.
- Two bases: 2 times 3.14 times 4 equals 25.12 square centimetres.
- The lateral face starts with 2 times 3.14, which equals 6.28.
- Then 6.28 times 2 equals 12.56, and 12.56 times 10 equals 125.6 square centimetres.
- Add 25.12 and 125.6 to get 150.72 square centimetres.



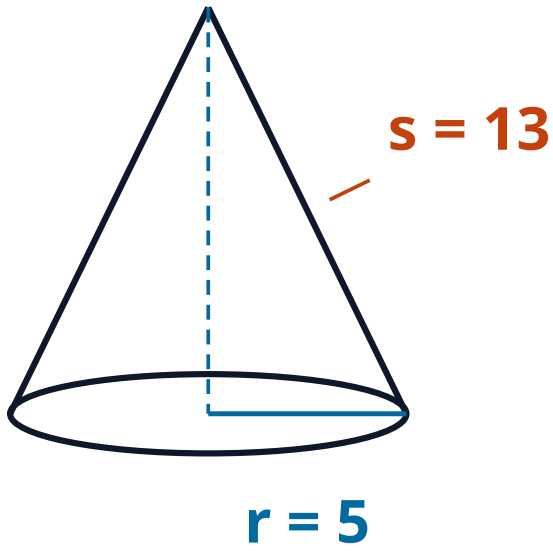
A hopper with a 3-4-5 slant

- A right cone with radius 3 and height 4 has slant height 5.
- Three squared is 9 and four squared is 16.
- Nine plus 16 equals 25, which is 5 squared.
- The surface area is πr^2 plus $\pi r s$.
- That is 9 pi plus 15 pi, which is 24 pi.
- With pi taken as 3.14, 24 times 3.14 equals 75.36 square units.



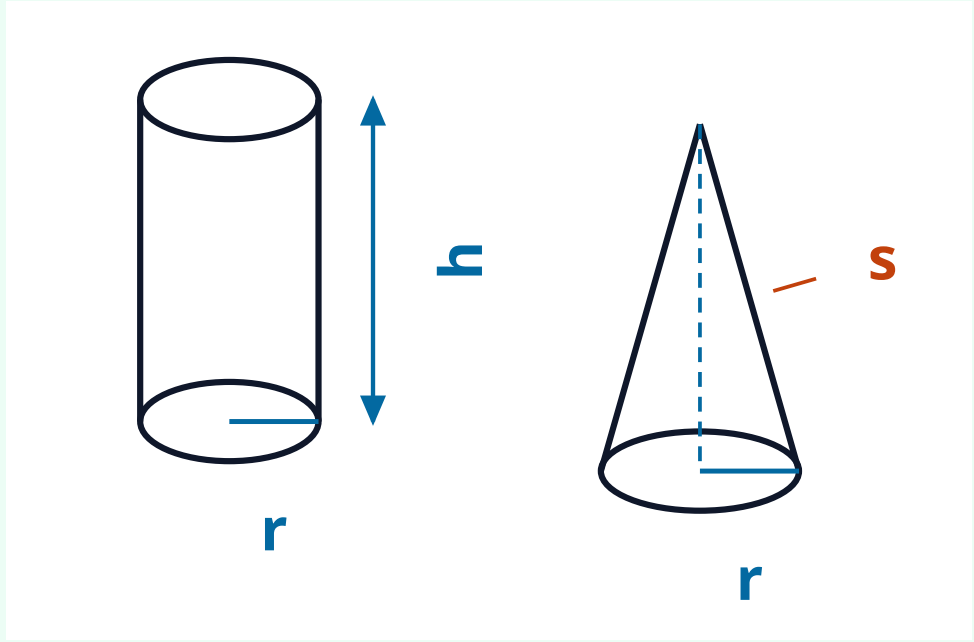
A larger cone and a sense check

- The base uses 5 times 5, which equals 25, so that part is 25π.
- The lateral face uses 5 times 13, which equals 65, so that part is 65π.
- Adding 25 and 65 equals 90, so the surface area is 90π.
- Multiplying 90 by 3.14 equals 282.6 square units.
- A sensible total is larger than one base alone.
- The duct total 2.1352 square metres is larger than one base of 0.1256 square metres.



Metal-order check

- State the radius you would substitute for a duct of diameter 0.4 metres, and the surface area 2.1352 square metres.
- State the slant you would use for radius 3 and vertical height 4, and the surface area 75.36 square units.



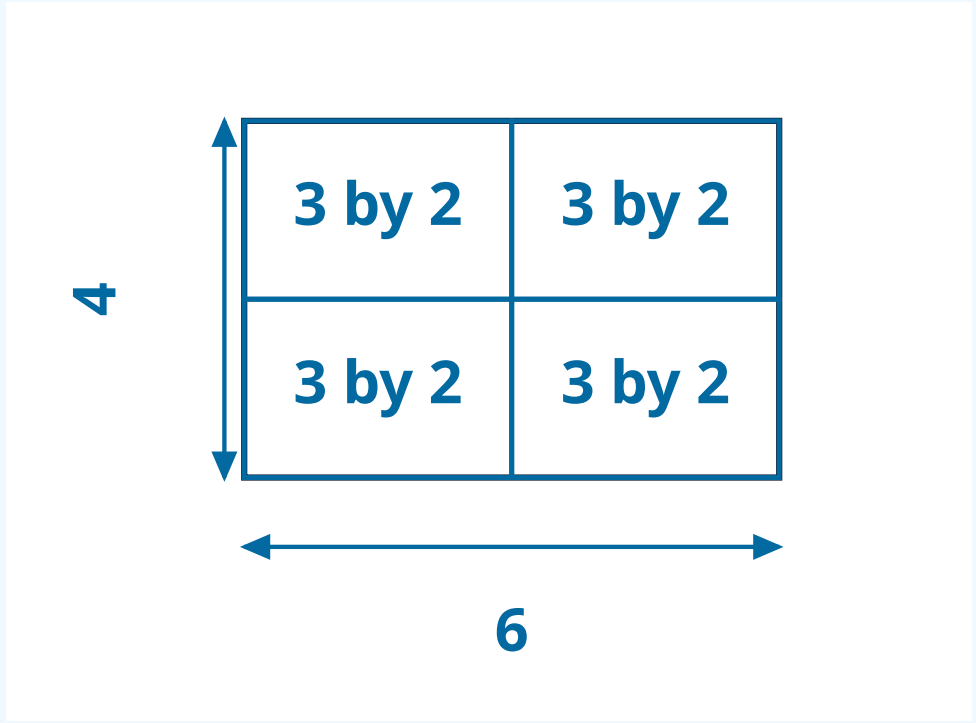
Mathematics 10-3

Unit G: Geometry and trigonometry

Packing puzzles and sliding tiles

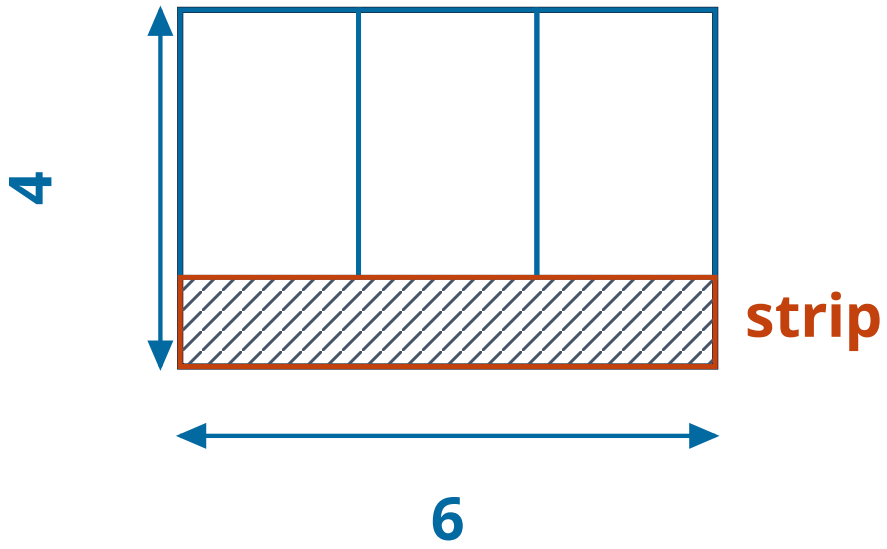
Four panels in a crate

- A 6 by 4 crate has to hold panels that are 3 by 2.
- Two panels fit along the 6, and two fit along the 4.
- That layout uses 4 panels and covers the crate.
- Turn every panel and the 4 side is no longer a whole number of panel edges.



Ways to work a packing puzzle

- Guess and check means you try one layout, then write what happened.
- A systematic list keeps every try.
- A drawing or a model shows the crate.
- Eliminating a try drops a layout that leaves a gap.
- Simplify means you solve a smaller crate first.
- Work backward means you start from a full crate and undo one panel.



Key terms

systematic list

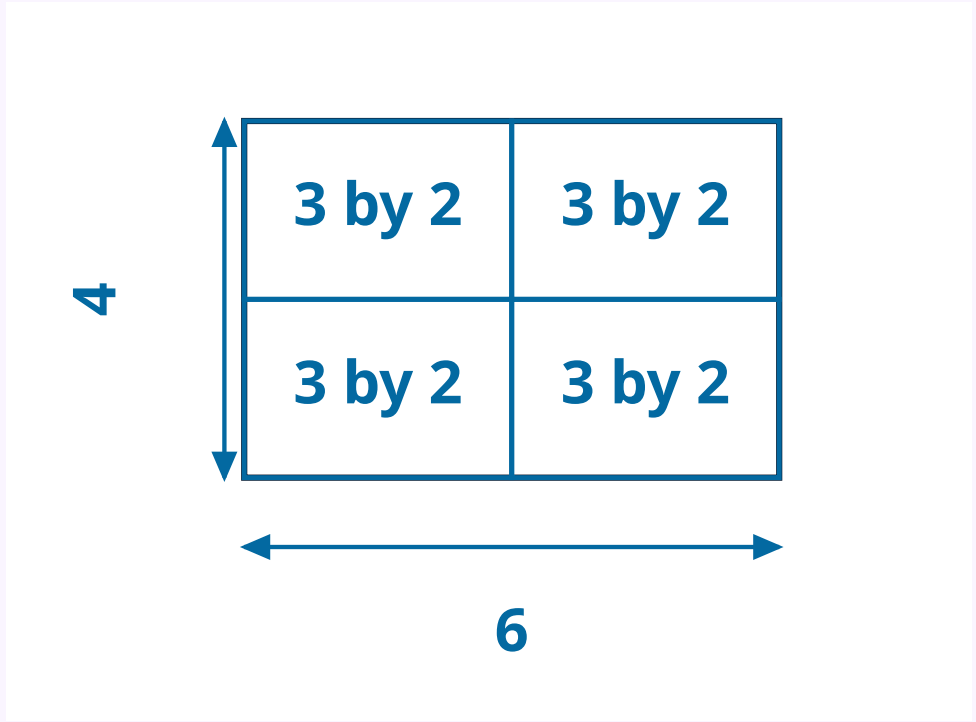
A written record of every try, kept in order, so the next try is a new move.

gap

The empty square in a sliding-tile tray. Tiles move by sliding into that empty square.

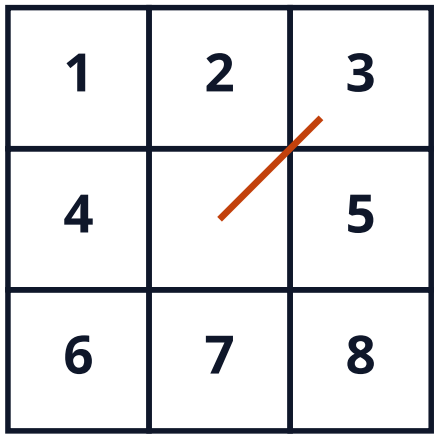
Area count and edge fit

- The crate area is 6 times 4, which is 24.
- Each panel is 3 times 2, which is 6.
- That 24 divided by 6 is 4 panels.
- Along 6 and 4, the panels sit as 3 plus 3 and 2 plus 2.
- That uses the 4 panels and the edges match.
- Turned panels ask 4 divided by 3.
- That split leaves a strip, so this orientation stays off the truck.



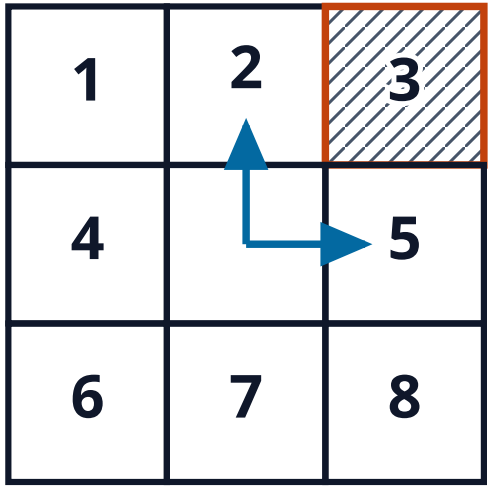
A lane of sliding tiles

- A 3 by 3 tray holds eight numbered tiles and one gap.
- A tile moves by sliding into the gap.
- The gap moves one square up, down, left, or right.
- A diagonal jump skips a square, so the tile would teleport.
- Finish the top row first, then treat the leftover 2 by 3 block as a smaller puzzle.



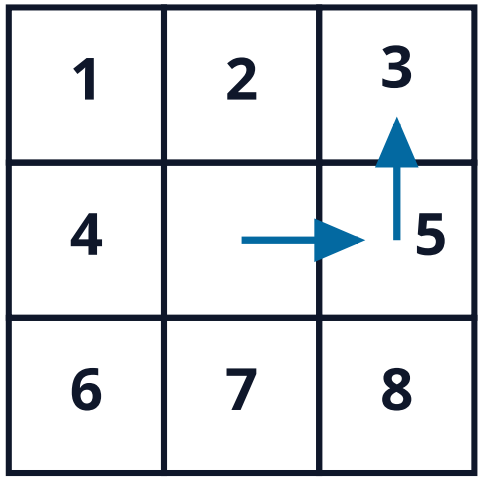
Block a corner and try again

- Cover the top-right square so no tile can land there.
- The goal is still a finished top row of the open squares.
- Work backward from that finished row: the gap has to sit just under the row before the last slide.
- Write the new rule in one sentence so a partner can try your variation.



Correct the written solution

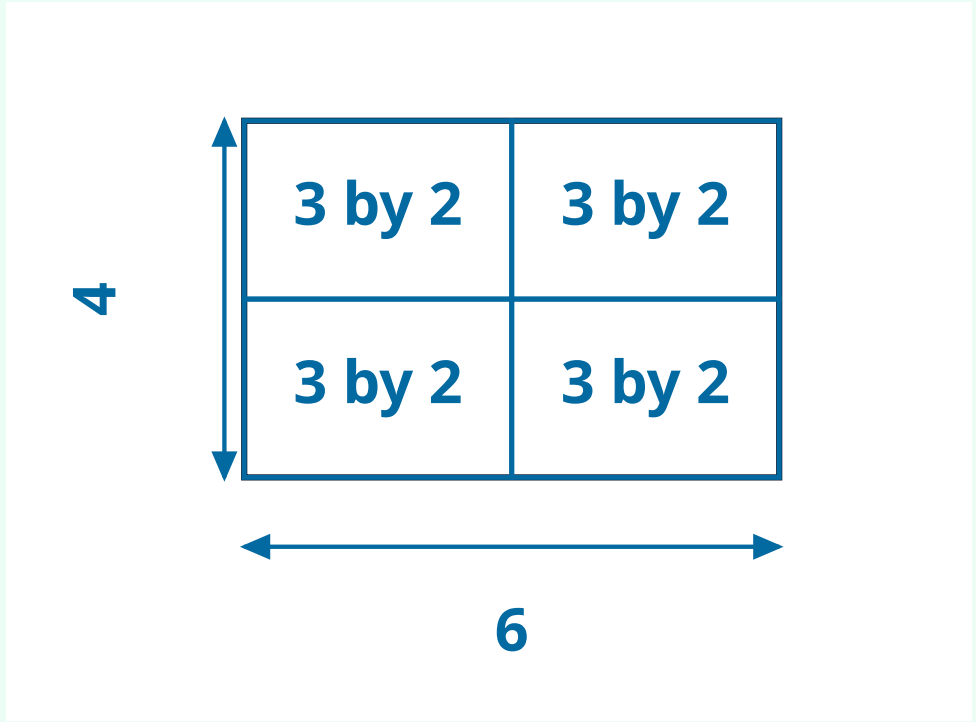
- A classmate's list says the gap moved from the middle square straight to a corner.
- The middle square touches the corner only at a point.
- The gap's next square has to share a full side.
- The fix is two slides: one to the side, then one up or down into the corner.



side, then up

Crate and tray check

- Tell how many 3 by 2 panels cover a 6 by 4 crate, and which way they face.
- Name the error in a list that slides the gap diagonally, and give the two-slide fix.



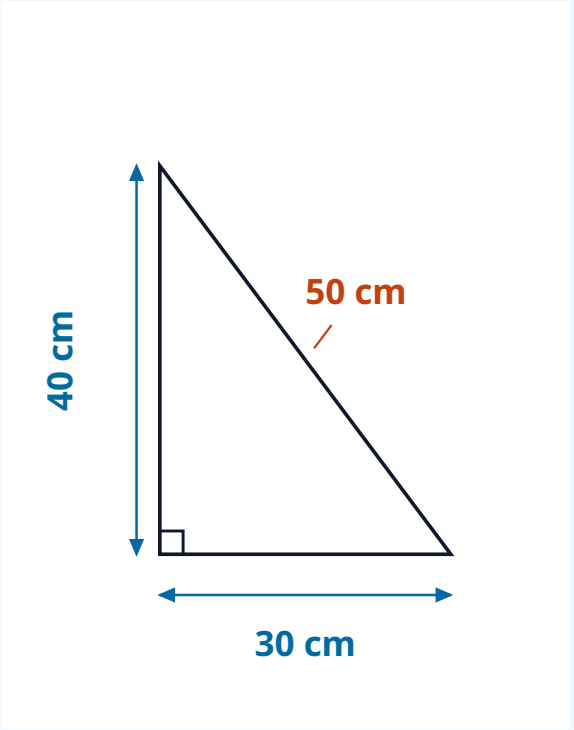
Mathematics 10-3

Unit G: Geometry and trigonometry

Right triangles and square corners

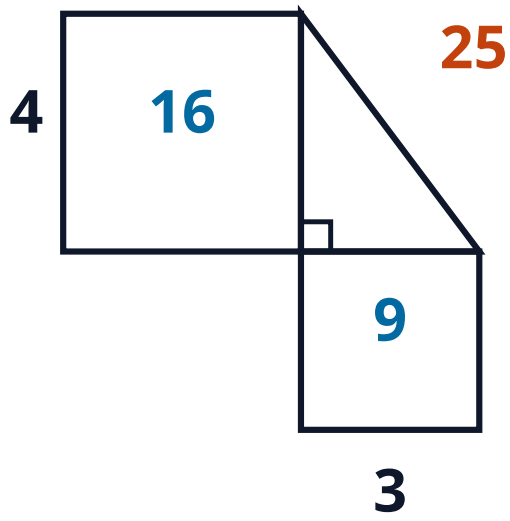
A corner that has to be square

- A framer marks 30 cm along one wall and 40 cm along the next wall.
- The diagonal between those marks should be 50 cm.
- The marks 30, 40, and 50 are 3, 4, and 5 each multiplied by 10.
- Builders have used a 3-4-5 rope to set a square corner, and a welder uses the same check on a frame.



Squares on the three sides

- The hypotenuse is the longest side, across from the right angle.
- Call the shorter sides a and b, and the hypotenuse c.
- The theorem says a squared plus b squared equals c squared.
- Squared means the number times itself.
- A 3-4-5 triangle gives 9 plus 16, which equals 25.
- A 2-3-4 triangle gives 4 plus 9, which is 13, while 4 squared is 16.



Key terms

hypotenuse

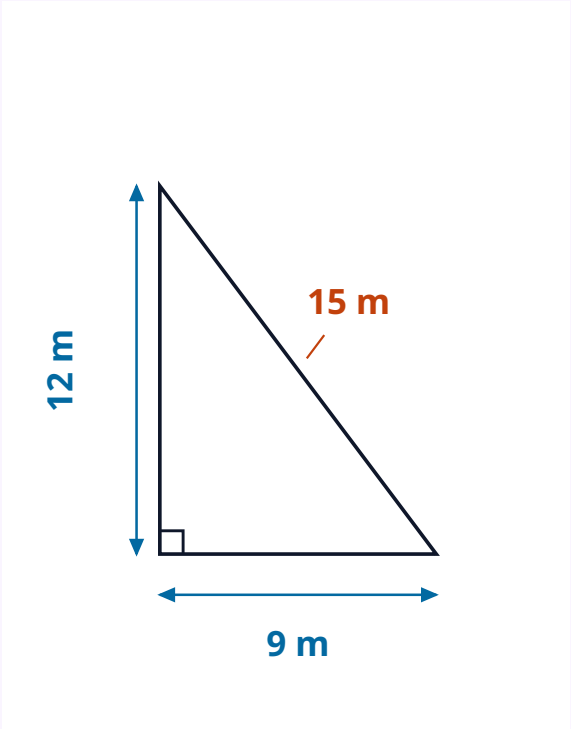
The longest side of a right triangle, the side across from the right angle. (c)

Pythagorean theorem

On a right triangle, the squares of the two shorter sides add to the square of the hypotenuse. ($a^2 + b^2 = c^2$)

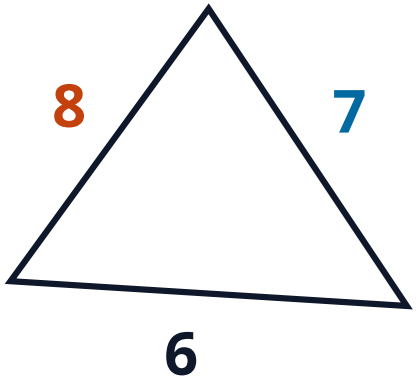
When the squares agree

- If a squared plus b squared equals c squared, the angle opposite c is a right angle.
- Square 9 to get 81 and square 12 to get 144.
- Adding 81 and 144 equals 225.
- Squaring 15 gives 225, so the 9 m by 12 m diagonal is 15 m.
- Square 6 to get 36 and square 8 to get 64.
- Adding 36 and 64 equals 100.
- Squaring 10 gives 100, so the brace on an 8 m run is 10 m.



A triangle that fails the square test

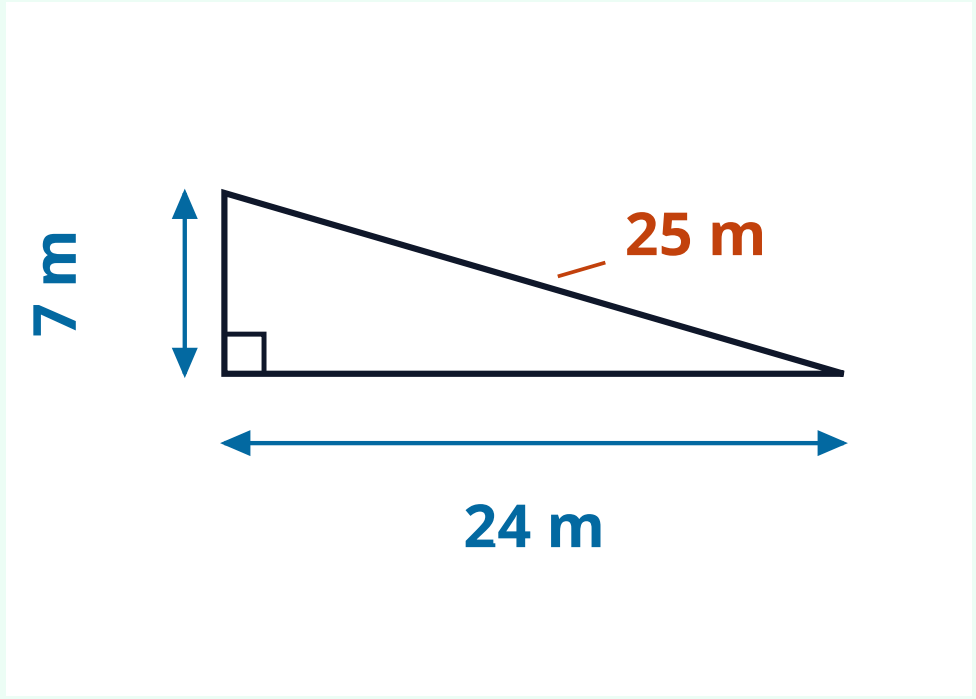
- Sides 6, 7, and 8 ask you to test the longest side, 8.
- Squaring 6 gives 36 and squaring 7 gives 49.
- Those add to 85.
- Squaring 8 gives 64.
- The totals 85 and 64 disagree, so this triangle fails the right-angle test.



$36 + 49 = 85$, and $8^2 = 64$

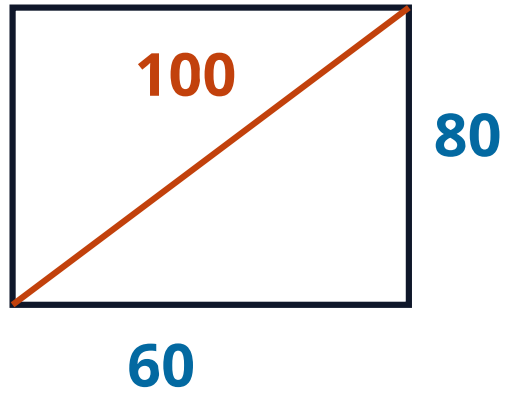
A 7 m by 24 m brace

- A gate opening is 7 m across the base and 24 m up the post.
- You need the diagonal brace.
- Squaring 7 gives 49 and squaring 24 gives 576.
- Those add to 625.
- Squaring 25 gives 625, so the brace is 25 m.



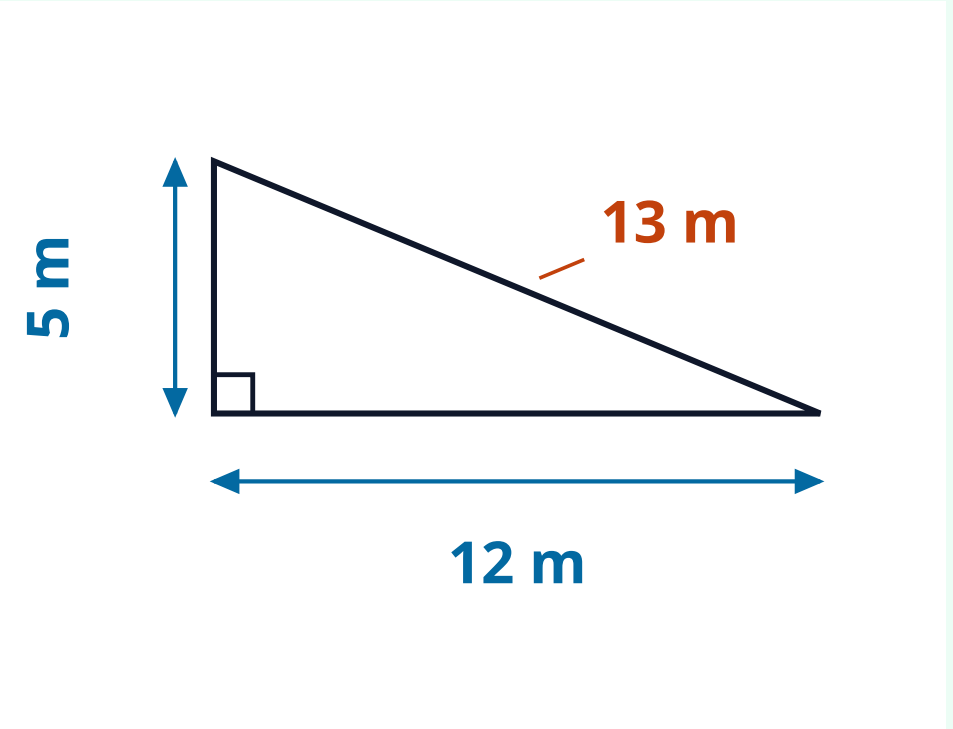
Square a corner on the job

- Multiply 3, 4, and 5 by the same number to get a larger square check.
- Times 10 gives 30, 40, and 50.
- If the measured diagonal matches 50 cm, the corner between the 30 cm and 40 cm marks is a right angle.
- A parallelogram with a 3-4-5 corner and equal diagonals is a rectangle.
- The square corner is what makes the frame true.



Square-corner check

- Find the diagonal of a 9 m by 12 m frame.
- Say what 36 plus 49 versus 64 tells you about a 6-7-8 triangle.



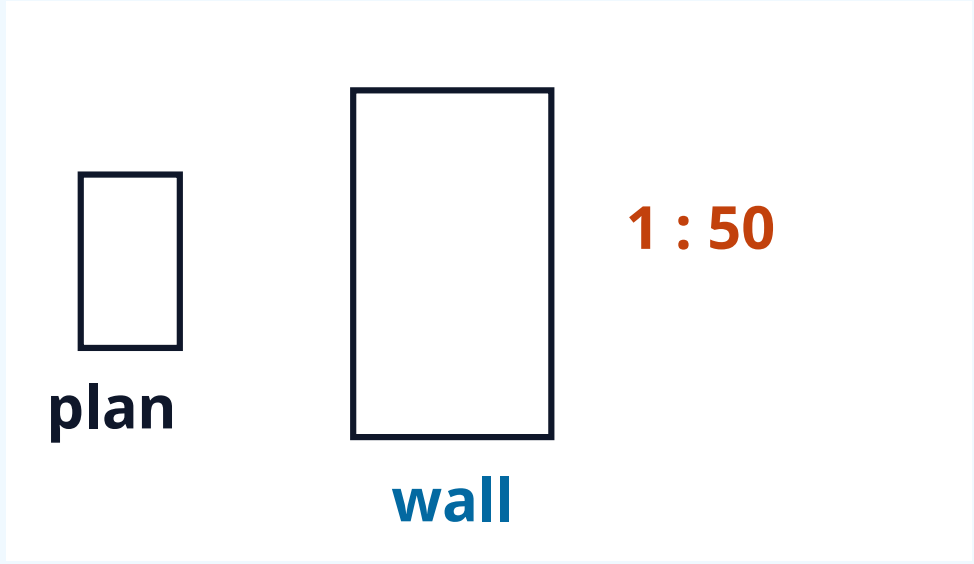
Mathematics 10-3

Unit G: Geometry and trigonometry

Similar polygons and scale drawings

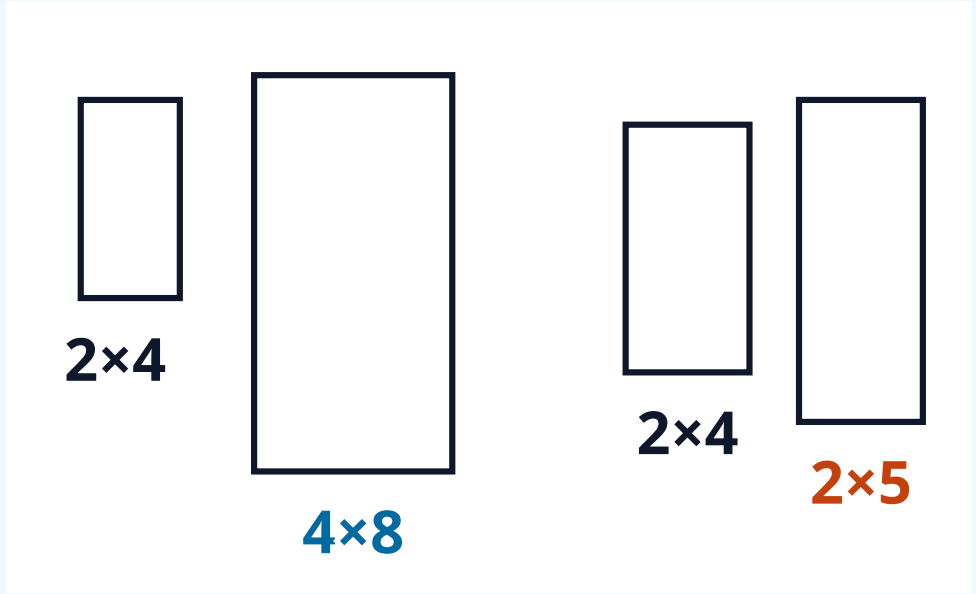
A wall on a site plan

- A site plan uses the scale 1 to 50.
- A wall that is 8.0 metres on the ground is 16 centimetres on the paper.
- The paper shape and the real wall share the same angles.
- Every real length is 50 times the paper length.



Angles match and sides stay in ratio

- Similar polygons have equal matching angles and one constant ratio between matching sides.
- A 2 by 4 rectangle and a 4 by 8 rectangle share right angles, and each side of the larger one is double.
- A 2 by 4 rectangle and a 2 by 5 rectangle share right angles, while the side ratios are 1 and 2.5.
- The similarity test fails.



Key terms

similar polygons

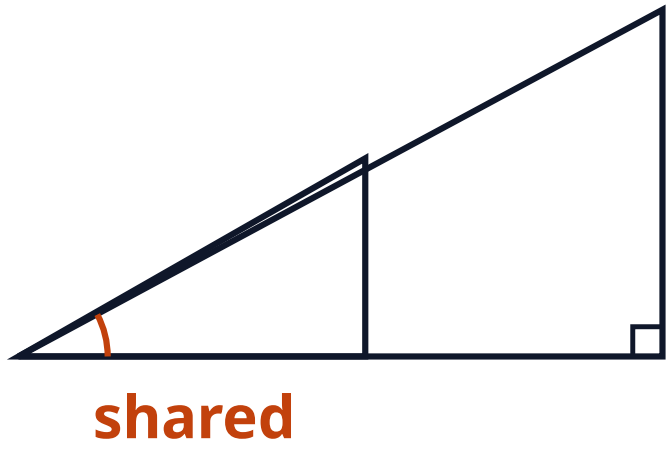
Polygons whose matching angles are equal and whose matching sides stay in one constant ratio.

scale factor

The number you multiply every side by when you draw a larger or smaller copy with the same angles. (1:50)

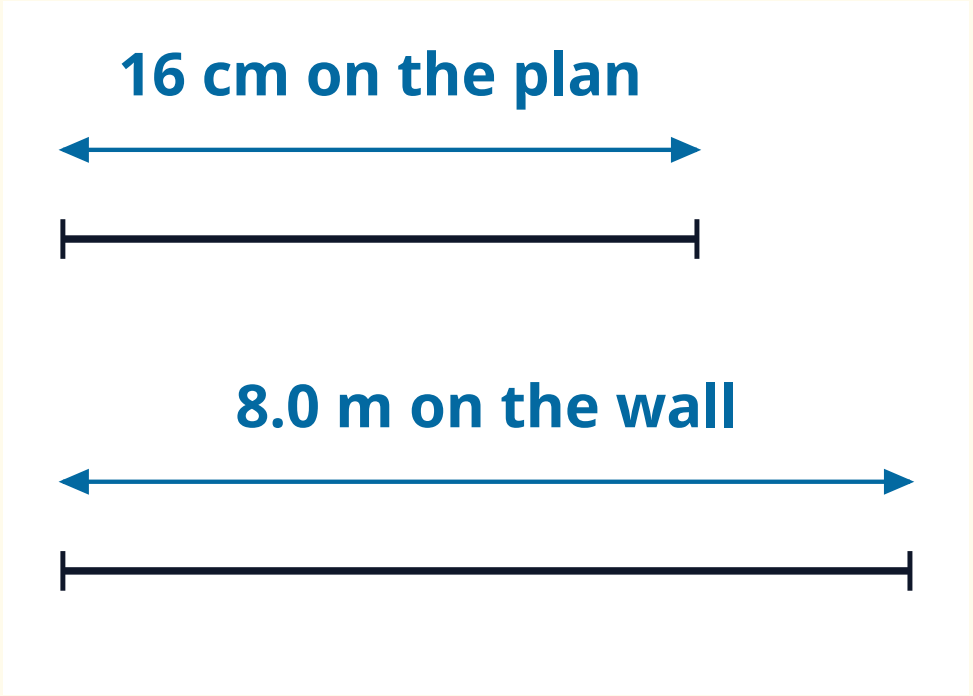
Triangles that share an acute angle

- Two right triangles that share an acute angle also share the right angle, so the third angles match too.
- Matching angles on a triangle force the sides into one ratio.
- A 3-4-5 triangle and a 9-12-15 triangle have ratio 3.
- On a polygon with more than three sides, matching angles still need a side-ratio check.
- Rectangles show why.



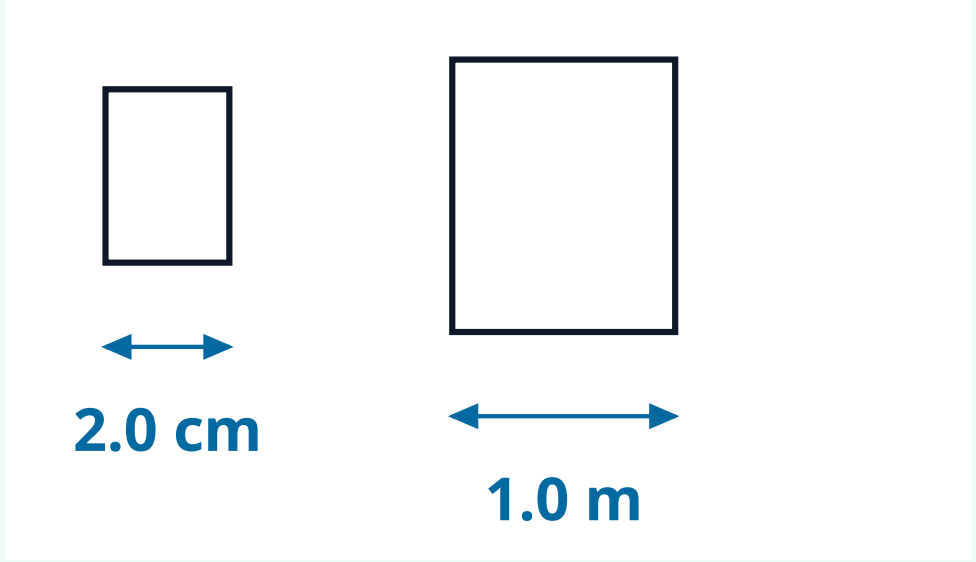
Wall length from a plan

- The scale 1 to 50 means 1 centimetre on the plan stands for 50 centimetres on the ground.
- The wall is 8.0 metres, which is 800 centimetres.
- Dividing 800 by 50 gives 16, so the wall is 16 centimetres on the plan.



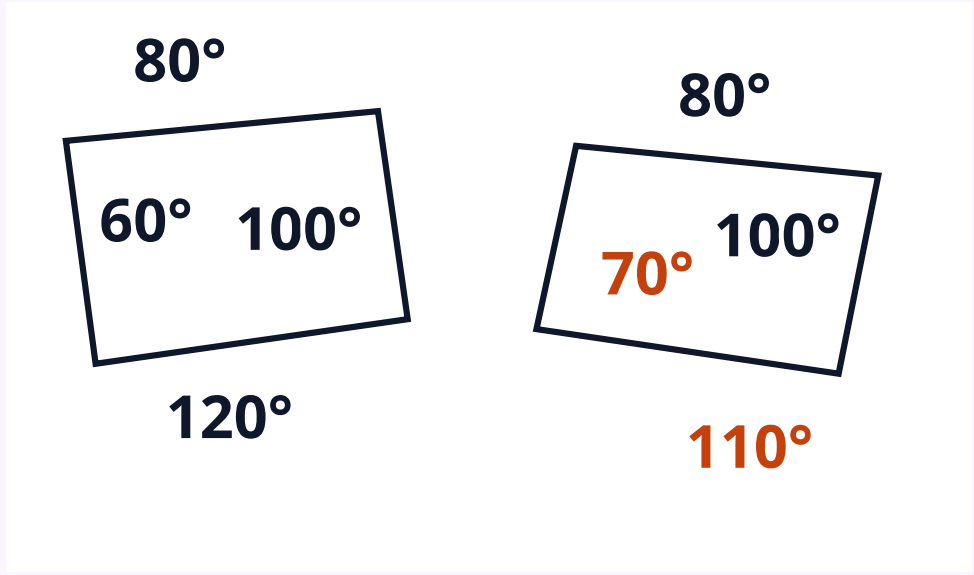
A window on the same plan

- A window measures 2.0 centimetres on the 1 to 50 plan.
- Multiply 2.0 by 50 to get 100 centimetres on the ground.
- That 100 centimetres is 1.0 metre, so the real window is 1.0 metre wide.



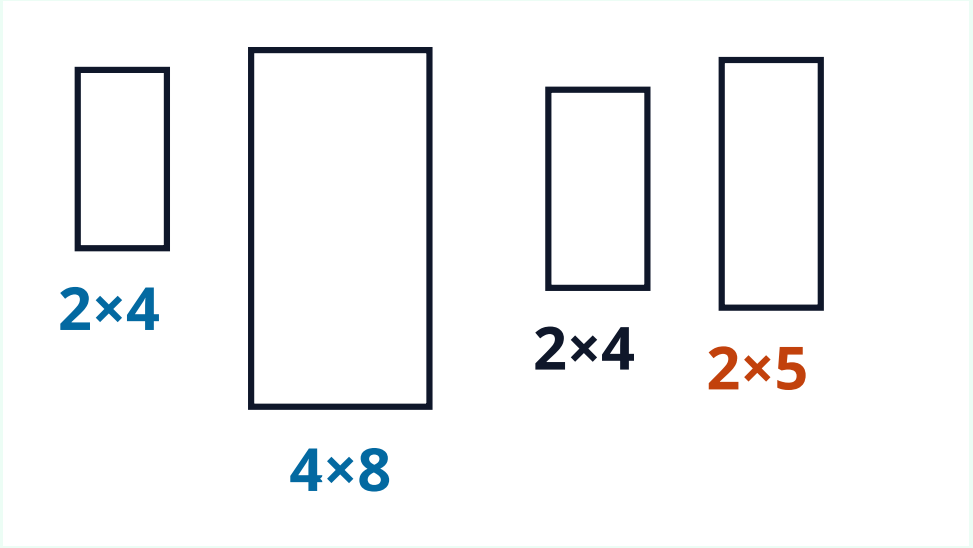
A quadrilateral angle check

- One quadrilateral has angles 80, 100, 120, and 60 degrees.
- A second has angles 80, 100, 110, and 70 degrees.
- The 120 degree angle has no match in the second polygon, so the angle test already fails.



Similarity check

- Explain why the 2 by 4 rectangle and the 4 by 8 rectangle pass the side-ratio test.
- Explain why the 2 by 5 rectangle fails against the 2 by 4.
- Find the plan length of an 8.0 metre wall at a scale of 1 to 50.



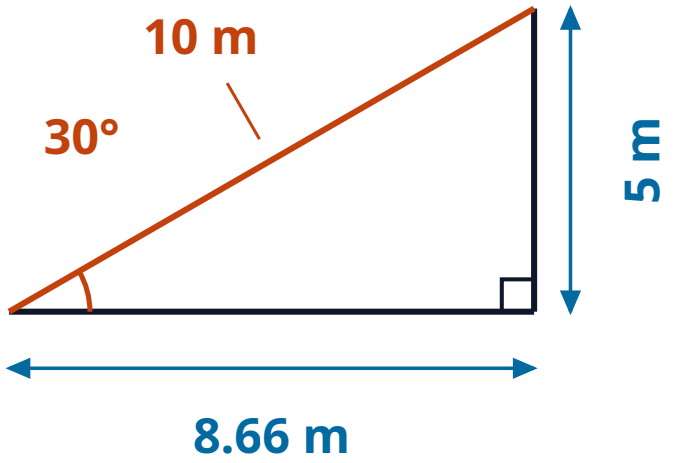
Mathematics 10-3

Unit G: Geometry and trigonometry

Primary trigonometric ratios

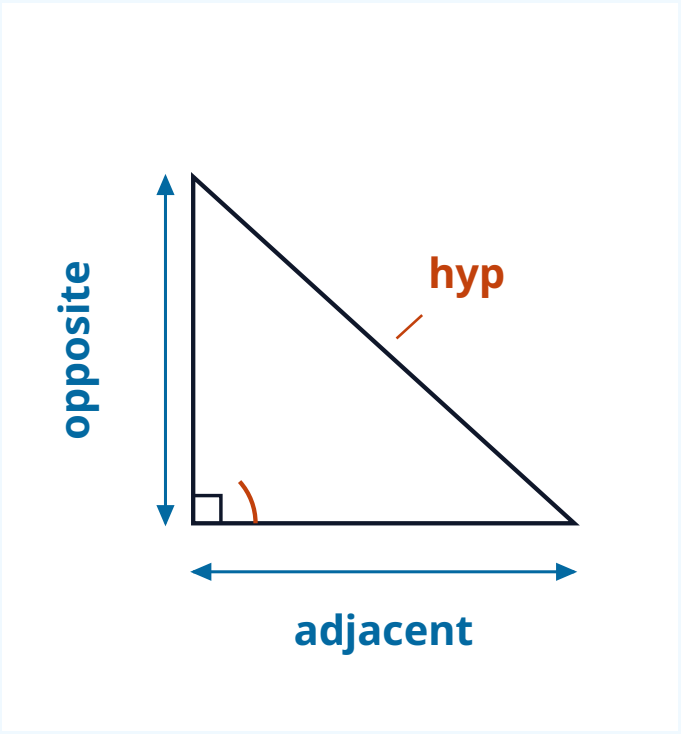
A ladder at 30 degrees

- A 10 metre ladder leans against a wall at 30 degrees to the ground.
- The height it reaches is 5 metres.
- Sine of 30 degrees is 0.5, so height equals 10 times 0.5.
- The same angle on a longer ladder keeps that 0.5 ratio.



Three ratios from similar right triangles

- Opposite means the leg across from the angle.
- Adjacent means the leg beside the angle.
- Hypotenuse means the longest side.
- Tangent equals opposite divided by adjacent.
- Sine equals opposite divided by hypotenuse.
- Cosine equals adjacent divided by hypotenuse.
- For 30 degrees, sine is 0.5, cosine is 0.866, and tangent is 0.577.
- For 45 degrees, sine and cosine are both 0.707 and tangent is 1.



Key terms

opposite

The leg across from the acute angle you are using. (opp)

adjacent

The leg that sits beside the acute angle and forms that angle with the hypotenuse. (adj)

sine

The ratio of the opposite leg to the hypotenuse for a fixed acute angle. (sin = opp/hyp)

cosine

The ratio of the adjacent leg to the hypotenuse for a fixed acute angle. (cos = adj/hyp)

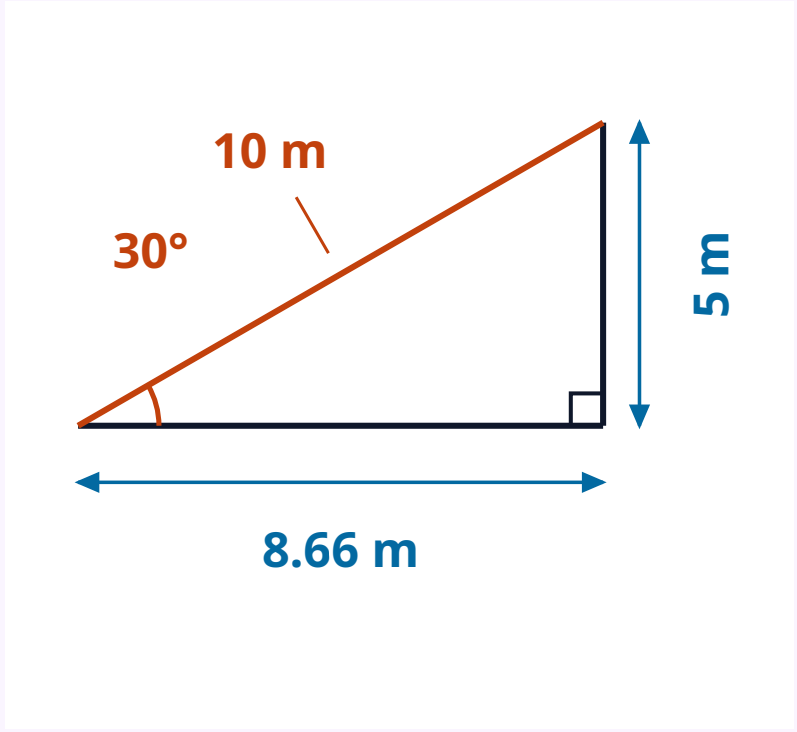
Key terms

tangent

The ratio of the opposite leg to the adjacent leg for a fixed acute angle. (tan = opp/adj)

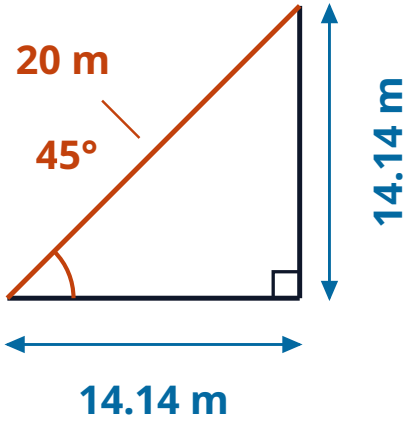
Height and base of the 10 metre ladder

- The height is opposite the 30 degree ground angle, so height equals hypotenuse times sine.
- Height equals 10 times 0.5, which is 5 metres.
- The base is adjacent, so base equals 10 times cosine of 30 degrees.
- That is 10 times 0.866, which is 8.66 metres.



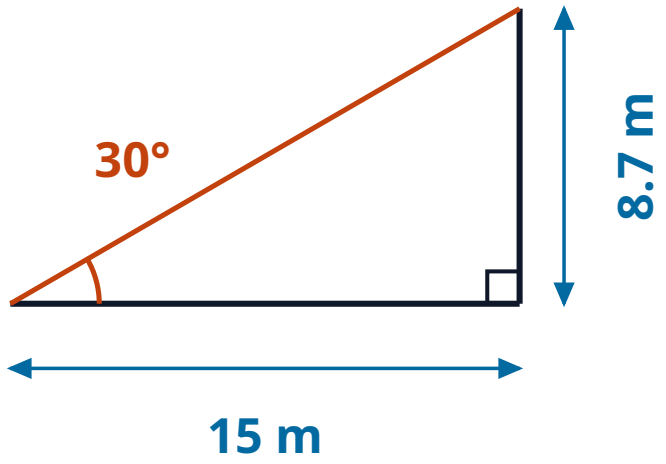
A 20 metre ladder at 45 degrees

- At 45 degrees, sine and cosine are both 0.707, and the two legs match.
- Height equals 20 times 0.707, which is 14.14 metres.
- The base equals 20 times 0.707 as well, so the base is also 14.14 metres.



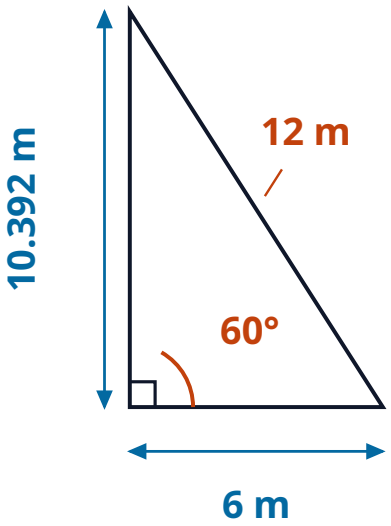
A pole from the ground

- You stand 15 metres from a pole.
- The angle of elevation to the top is 30 degrees.
- The 15 metre ground distance is the adjacent side.
- The pole height is the opposite side.
- Height equals 15 times tangent of 30 degrees.
- Tangent of 30 is 0.577, so 15 times 0.577 equals 8.655 metres, about 8.7 metres.



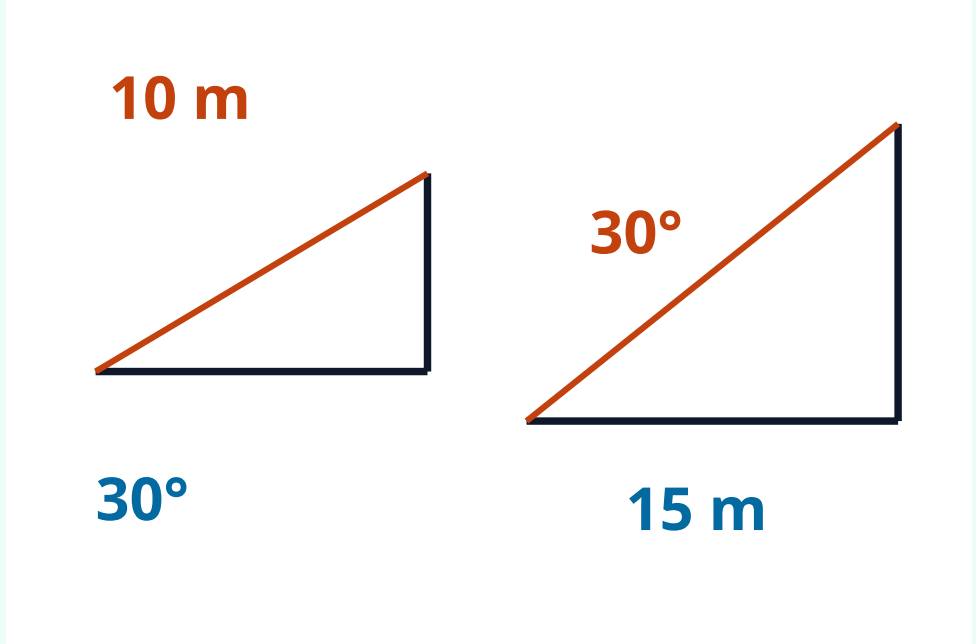
A 12 metre hypotenuse at 60 degrees

- Sine of 60 degrees is 0.866 and cosine of 60 degrees is 0.5.
- The opposite side equals 12 times 0.866, which is 10.392 metres.
- The adjacent side equals 12 times 0.5, which is 6 metres.
- A sensible check is that both legs are shorter than the 12 metre hypotenuse.



Ratio check

- Find the wall height for a 10 metre ladder at 30 degrees to the ground.
- Find the pole height when you stand 15 metres away at an elevation of 30 degrees.
- Use tangent 0.577.



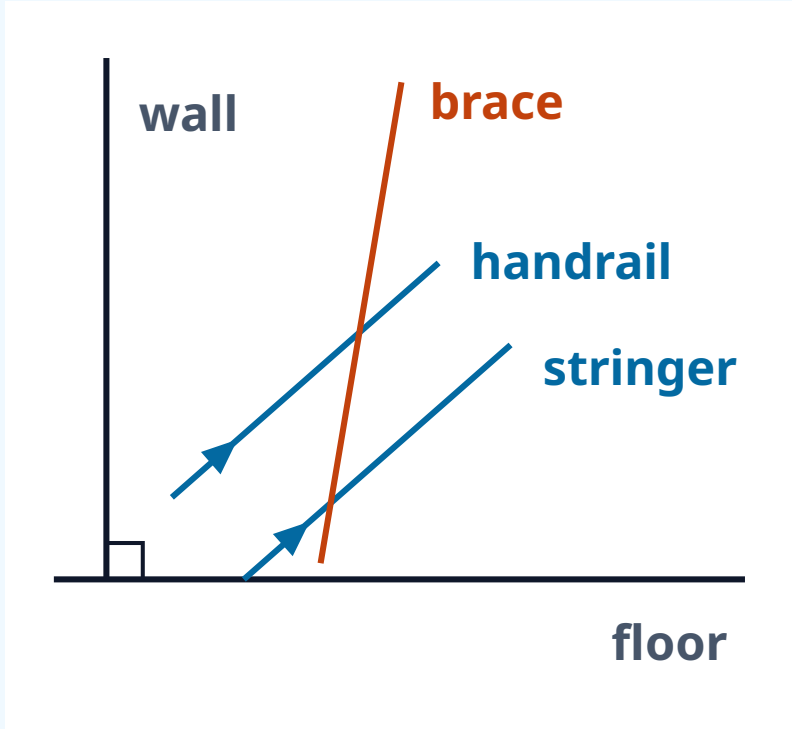
Mathematics 10-3

Unit G: Geometry and trigonometry

Parallel lines and transversals

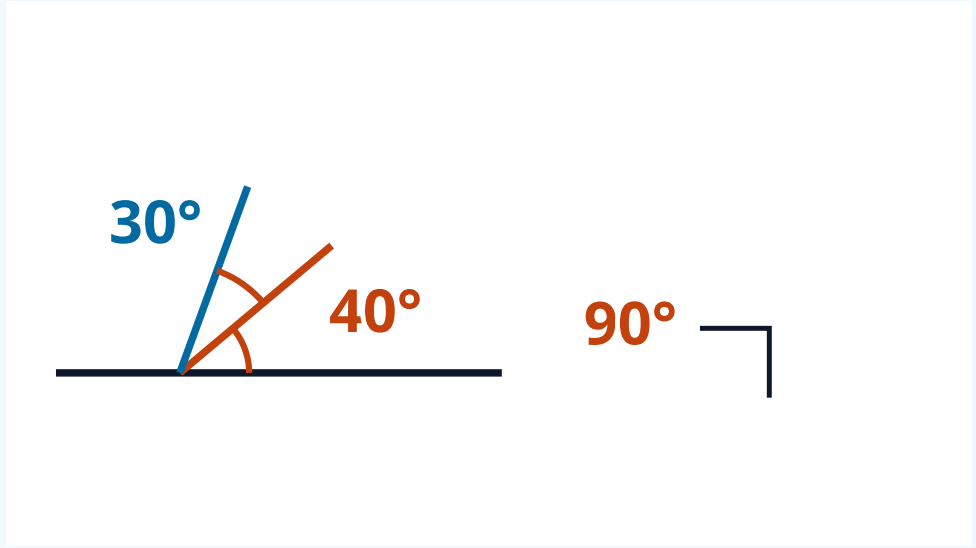
Parallel lines in a stair

- On a stair, the handrail runs the same way as the stringer, so those two lines are parallel.
- The wall meets the floor in a square corner, so the wall is perpendicular to the floor.
- A diagonal brace cuts across both lines.
- The brace is a transversal, a line that crosses two or more other lines.



Angle pairs on a crossing line

- Complementary angles add to 90 degrees.
- Supplementary angles add to 180 degrees.
- A transversal is the line that cuts across two others and makes a pair of angles at each crossing.
- When the two lines are parallel, corresponding angles match, and same-side interior angles add to 180 degrees.



Key terms

parallel lines

Lines that keep a steady gap, like a handrail and the stringer it follows. (||)

transversal

A line that crosses two or more other lines and creates pairs of angles at the crossings. (t)

corresponding angles

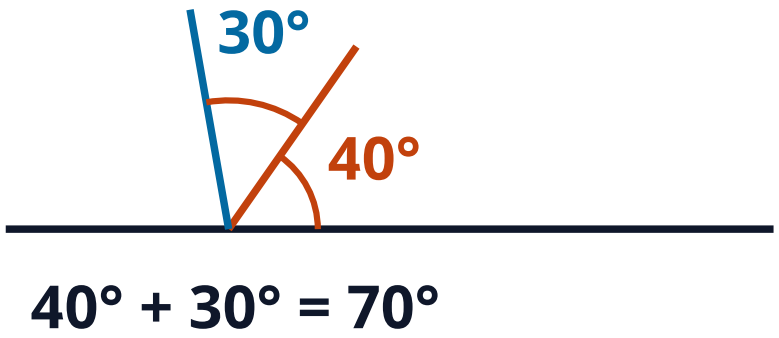
Angles that sit in the same corner at each crossing of a transversal, such as both upper-left angles. (F shape)

same-side interior angles

The two angles between the parallel lines and on the same side of the transversal. They add to 180 degrees. (sum 180)

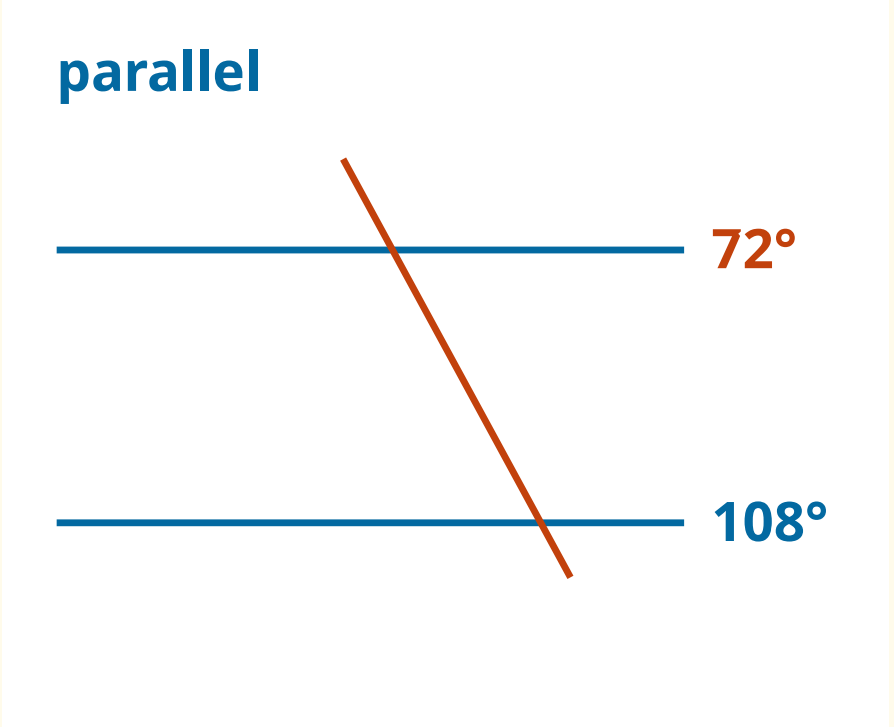
A 40 degree angle beside a 30 degree angle

- Two neighbouring angles measure 40 degrees and 30 degrees.
- Their sum is 70 degrees.
- A complementary pair needs 90 degrees and a supplementary pair needs 180 degrees, so this pair is a plain neighbouring pair.



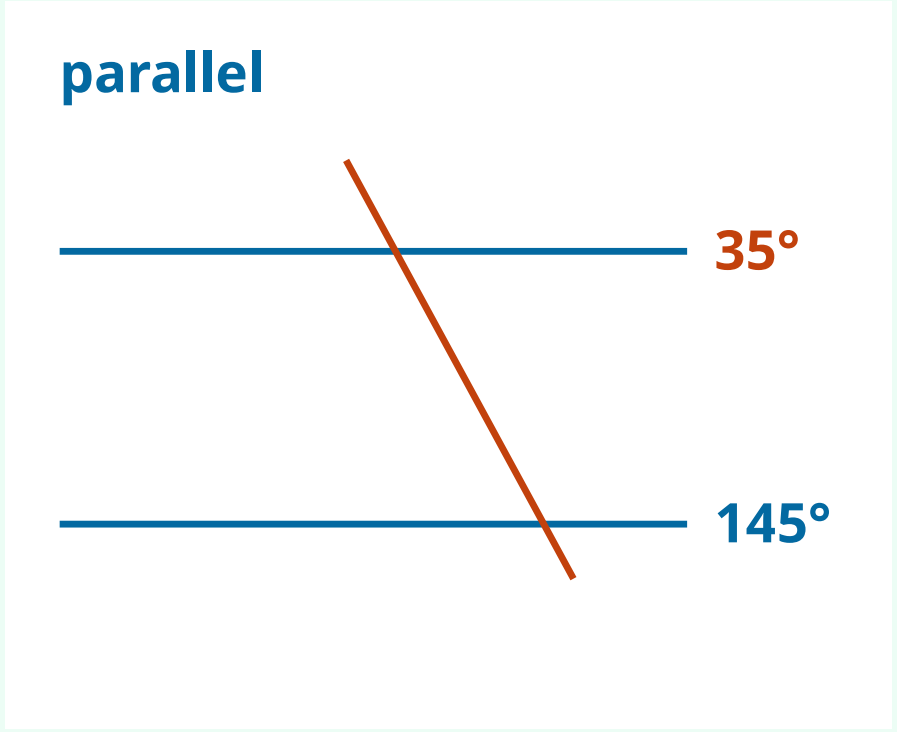
A 72 degree stair cut

- Two parallel rails are cut by a transversal, and one interior angle is 72 degrees.
- The corresponding angle and the alternate interior angle are also 72 degrees.
- The same-side interior angle is 180 minus 72, which is 108 degrees.



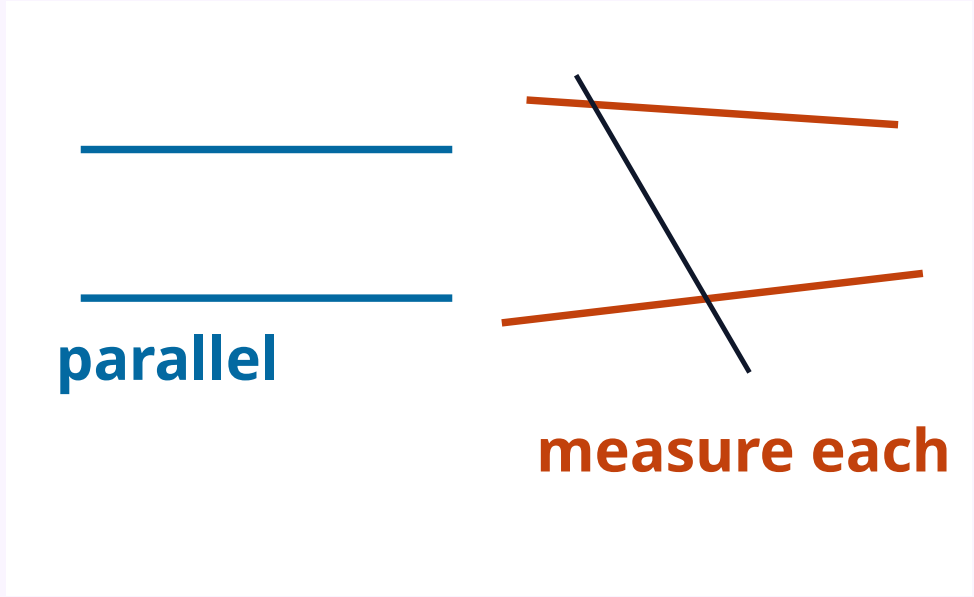
A 35 degree handrail

- A handrail is parallel to the stringer and a brace crosses it at 35 degrees.
- The angle that matches the 35 degree angle is 35 degrees.
- The same-side interior angle is 180 minus 35, which is 145 degrees.



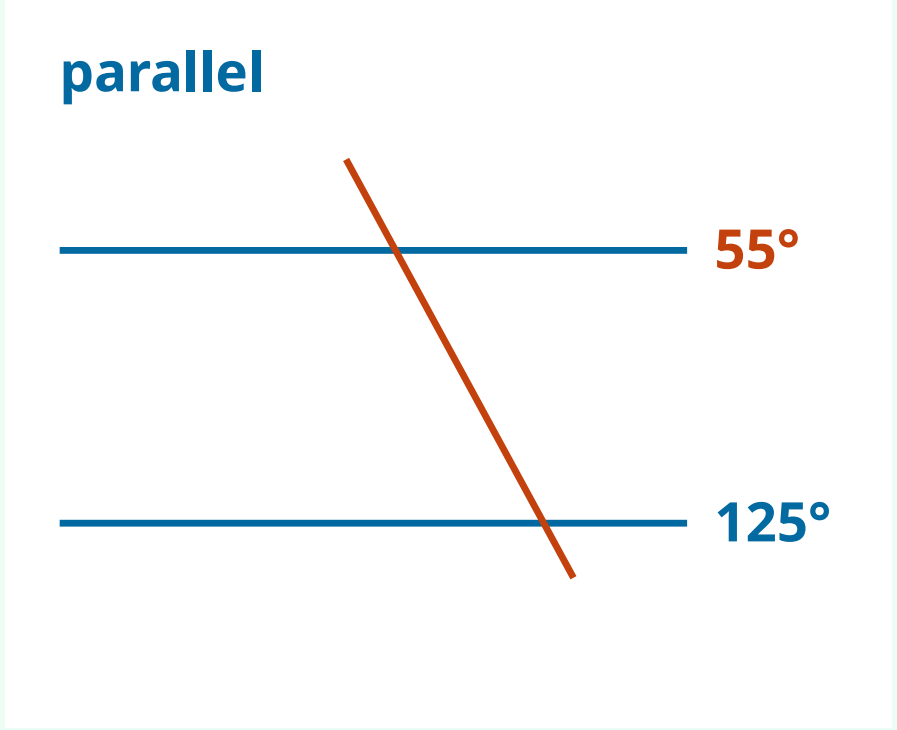
When the rails leave the parallel test

- Equal corresponding angles and 180 degree same-side pairs belong to a parallel pair of lines.
- A tilted rail fails that parallel test, so each angle at the crossing gets its own protractor reading.
- A wall that stands square to the floor is perpendicular.
- The right angle there is 90 degrees.



Angle check

- A stair stringer meets a vertical post at 55 degrees, and the stringer is parallel to the handrail.
- Find the same-side interior angle.
- Sort a wall and a floor, a handrail and a stringer, and a diagonal brace.



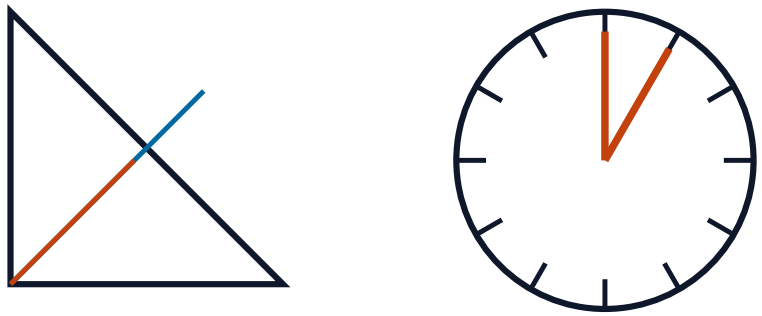
Mathematics 10-3

Unit G: Geometry and trigonometry

Drawing and bisecting angles

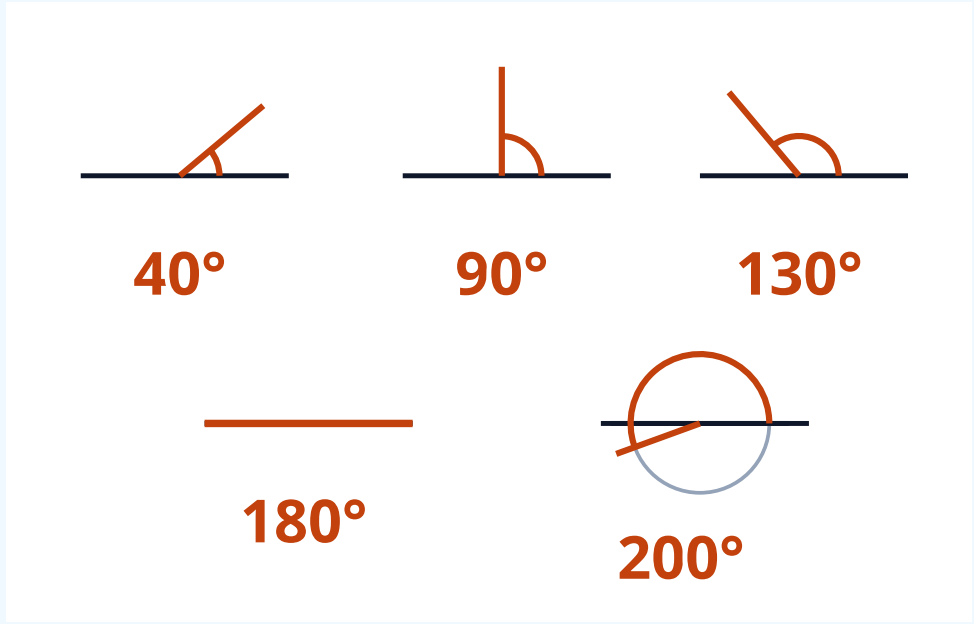
A paper corner and a clock

- A square corner of paper is 90 degrees.
- Fold it in half and the crease is 45 degrees.
- Fold that half again and the crease is 22.5 degrees.
- On a clock, the hour marks are 30 degrees apart, because 360 divided by 12 is 30.



Five angle names and a protractor

- An acute angle is under 90 degrees.
- A right angle is 90 degrees.
- An obtuse angle sits between 90 and 180.
- A straight angle is 180.
- A reflex angle sits between 180 and 360.



Key terms

acute angle

An angle smaller than a square corner, so its measure is under 90 degrees. (< 90)

obtuse angle

An angle larger than a square corner and smaller than a straight line, between 90 and 180 degrees. (90 to 180)

reflex angle

An angle that goes past a straight line and stops before a full turn, between 180 and 360 degrees. (180 to 360)

bisect

To cut an angle into two equal angles, such as a 70 degree angle cut into two 35 degree angles. (half)

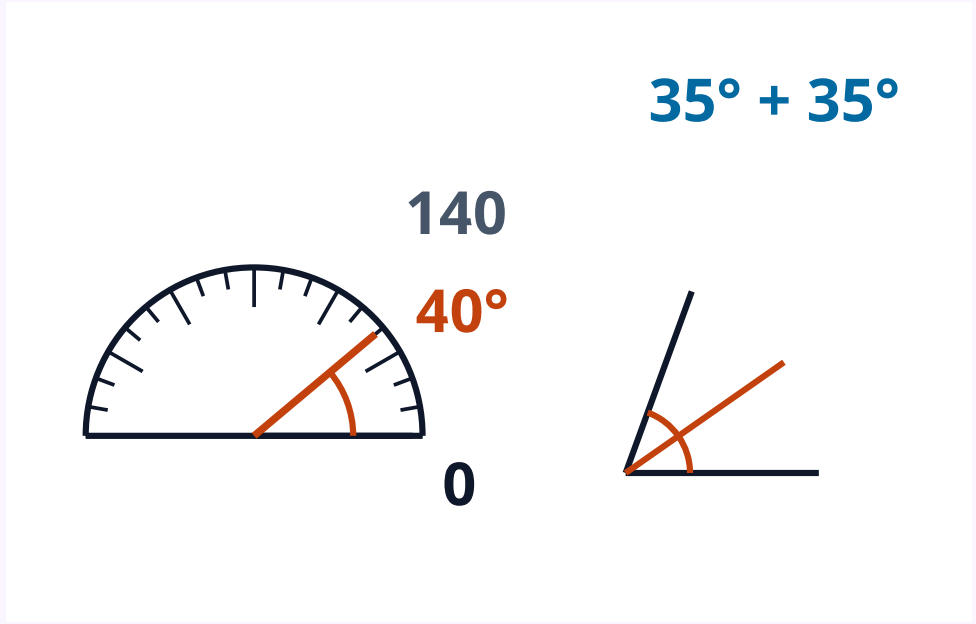
Key terms

protractor

A semicircular tool with two number scales, used by placing the centre on the vertex and reading the scale that starts at 0 on the first ray. (0 to 180)

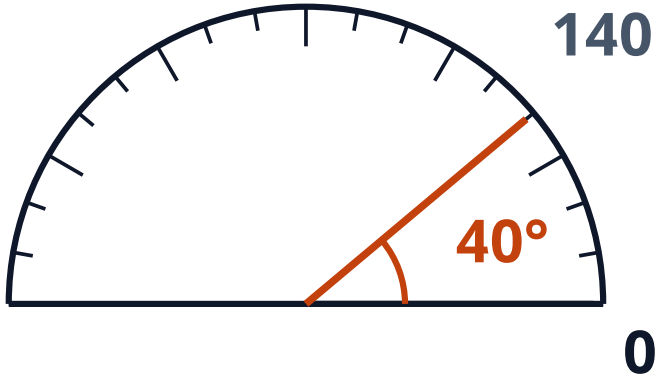
Protractors and angle bisectors

- Read the protractor scale whose 0 sits on the first ray.
- A 40 degree opening is recorded as 40 degrees.
- Bisect means cut into two equal angles.
- A 70 degree angle bisects into two 35 degree angles.



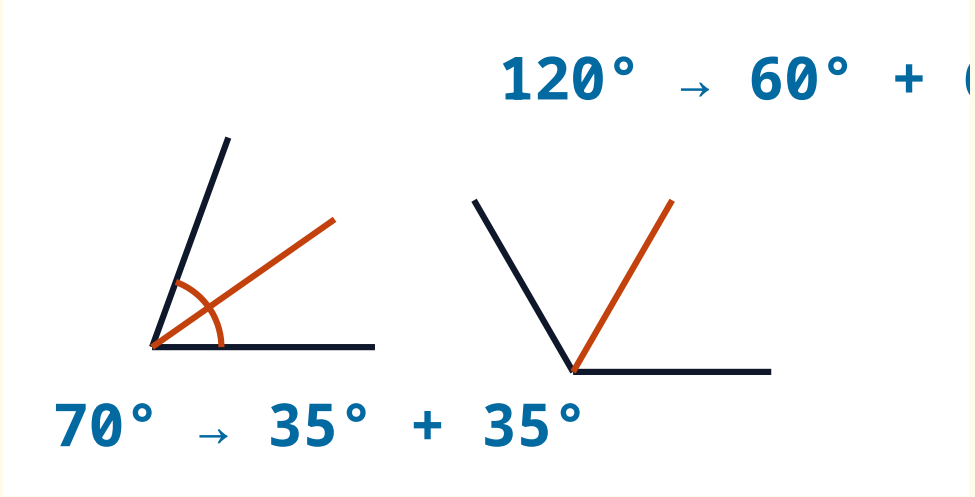
Estimating and measuring 40 degrees

- The referents are 22.5, 30, 45, 60, 90, and 180 degrees.
- A 40 degree opening is a little under the 45 degree paper fold, so the estimate is 40.
- The protractor confirms 40 degrees when the 0 of the scale sits on the first ray.



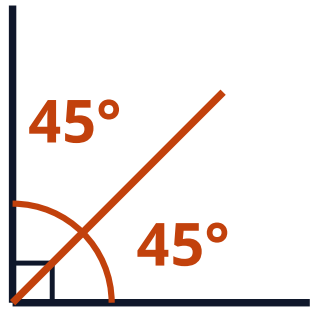
Bisect 70 degrees and 120 degrees

- Half of 70 is 35, so the bisector makes two 35 degree angles.
- Half of 120 is 60, so the bisector makes two 60 degree angles.
- A compass and straightedge, a protractor, or geometry software can each draw that bisector.



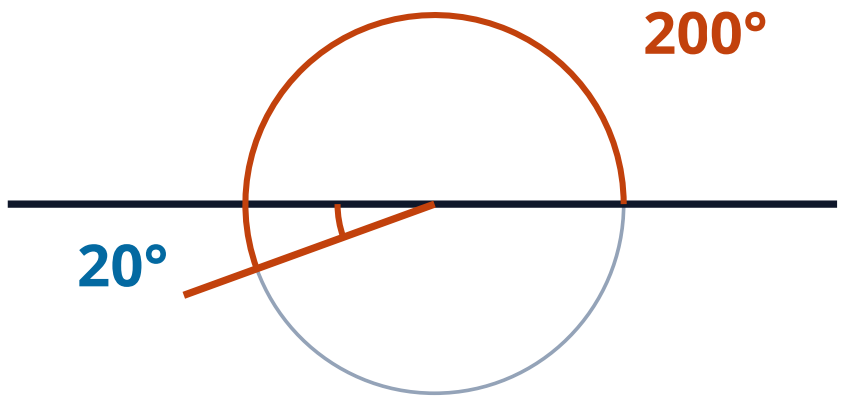
A 90 degree mitre

- A corner trim meets at 90 degrees, and the mitre cuts that corner in half.
- Half of 90 is 45, so each mitre piece is cut at 45 degrees.
- Check the cut against the paper fold.
- The fold and the saw mark should match.



A reflex angle past a straight line

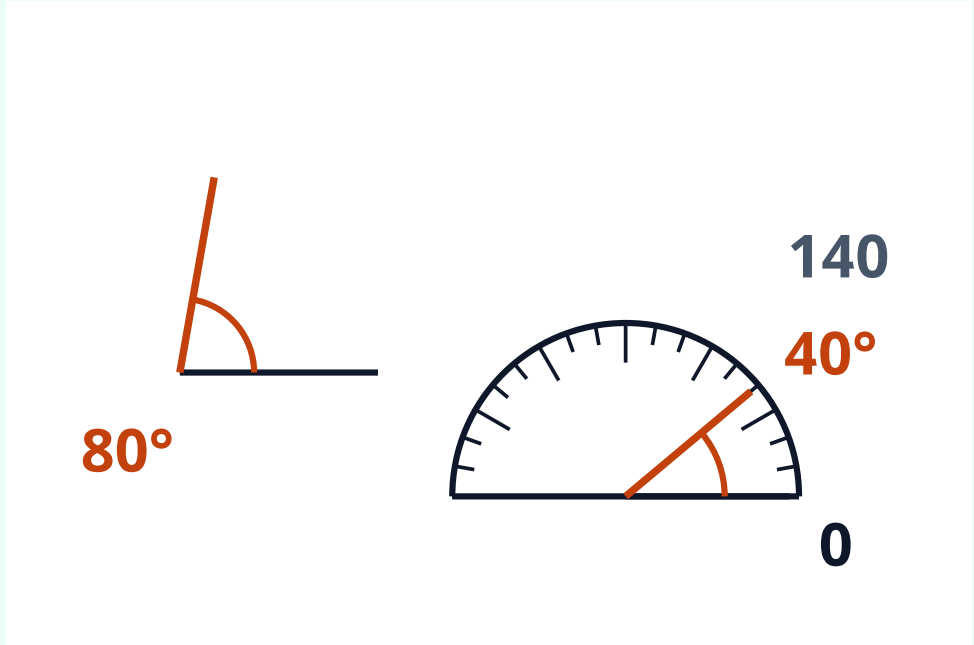
- A straight line is 180 degrees.
- About 20 degrees past that line is about 200 degrees.
- The reflex angle is the long way around, between 180 and 360.
- You can copy an angle with a protractor, a compass and straightedge, a carpenter square, or geometry software.



straight line plus about 20°

Angle check

- Bisect an 80 degree angle and name the two resulting angles.
- A drawn opening looks a little under a 45 degree fold.
- Which protractor reading fits, 40 degrees or 140 degrees?



Mathematics 10-3

Unit N: Money and income

Unit price and the better buy

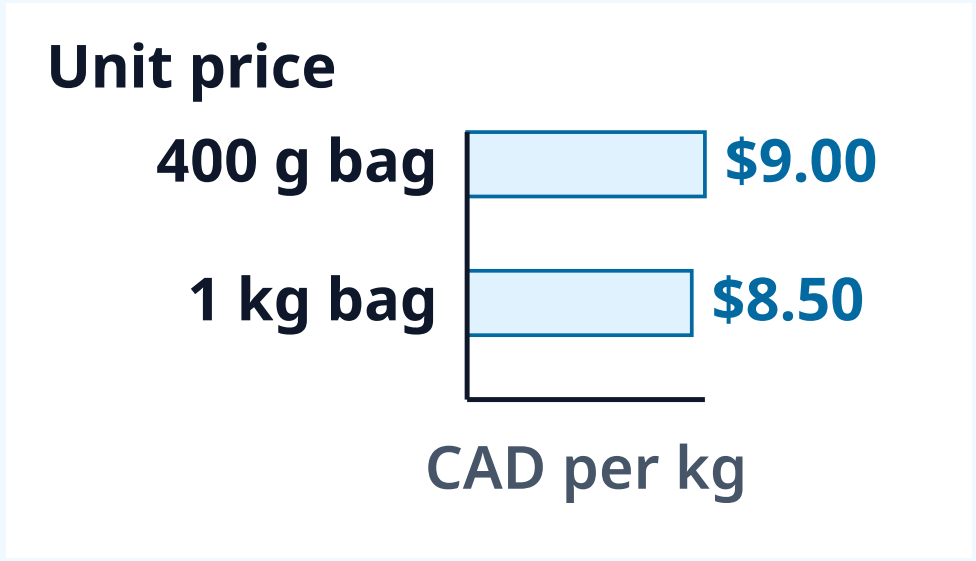
Two rice bags on a grocery shift

- A grocery clerk sets a 400 gram bag of rice at 3.60 dollars beside a 1 kilogram bag at 8.50 dollars.
- The smaller shelf price can look like the bargain until both bags are priced per kilogram.



Unit price and price change

- Unit price divides cost by quantity after both packages use one unit, such as dollars per kilogram or dollars per litre.
- A best buy weighs that unit price together with quality and with how much of the package will be used.
- Percent change uses the original price as the base, and a sales promotion may hide the same unit price inside a smaller printed unit.



Key terms

Unit price

The cost of one shared unit, found by dividing the price by the quantity. (price / quantity)

Best buy

The purchase that fits the unit price and the amount the buyer can actually use.

Percent change

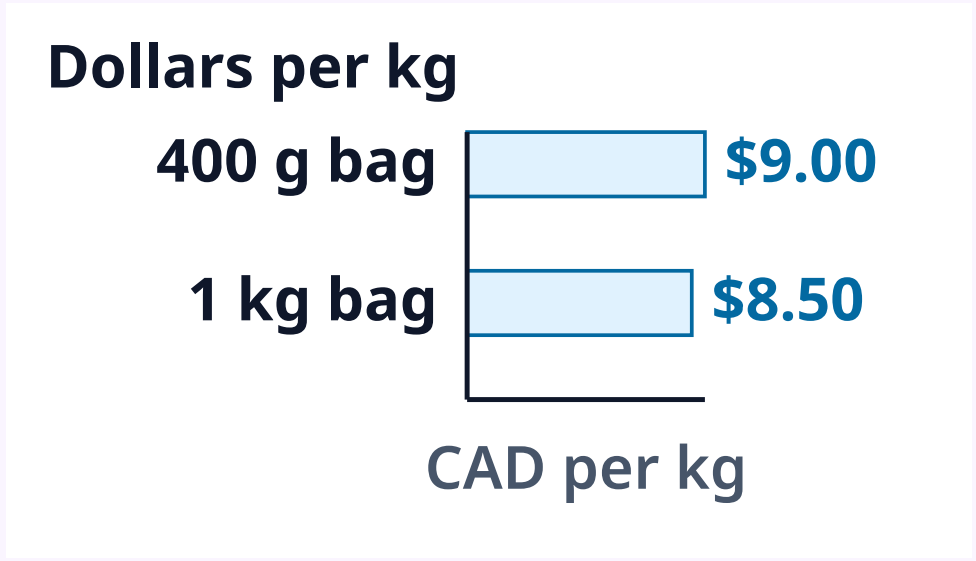
The price gap divided by the original price, then multiplied by 100 to name a percent increase or a percent decrease. ((new - original) / original × 100%)

Sales promotion

A store offer such as a multi-can deal, a percent-off tag, or a price written per 100 grams.

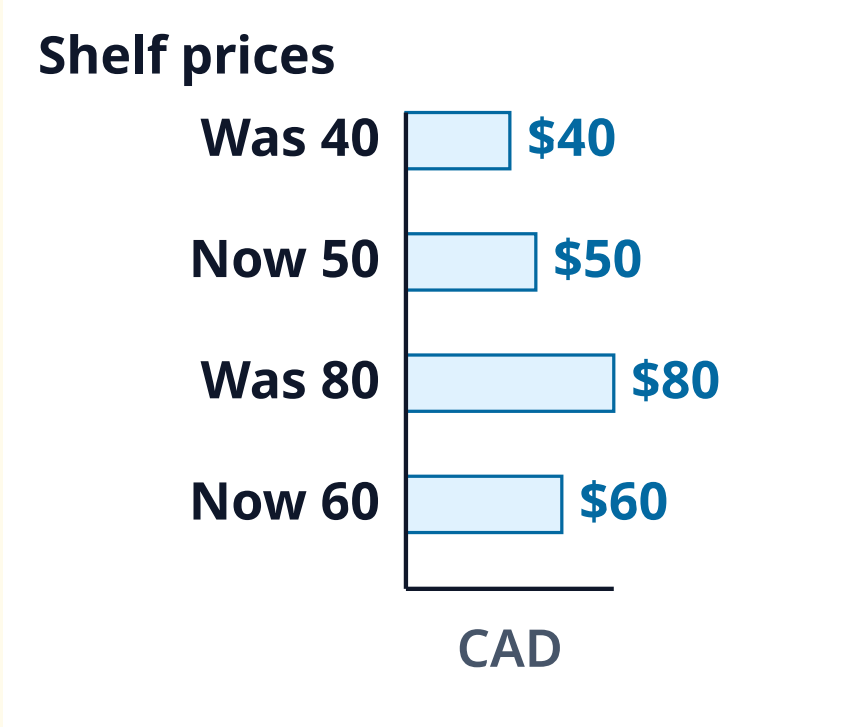
Rice bags in dollars per kilogram

- The 400 gram bag is 400 divided by 1000, which equals 0.4 kilograms, and 3.60 divided by 0.4 equals 9.00.
- The 1 kilogram bag is 8.50 divided by 1, which equals 8.50 dollars per kilogram, so the larger bag has the lower unit price.
- When the household can use only 400 grams before the rest spoils, the 3.60 dollars bag can still be the better buy.



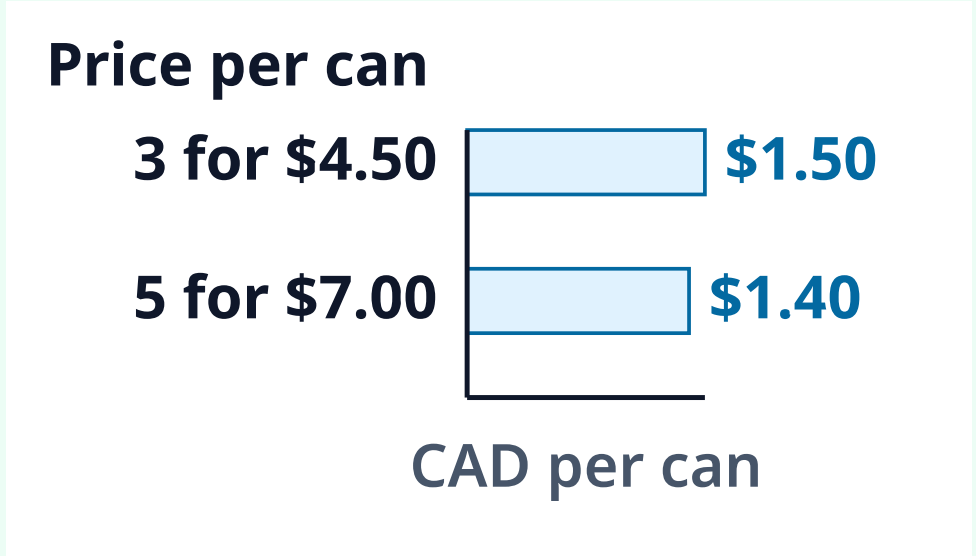
Percent increase and decrease

- Percent change equals the price gap divided by the original price, times 100%.
- From 40 dollars to 50 dollars the gap is 10, and 10 divided by 40 equals 0.25, a 25% increase.
- From 80 dollars to 60 dollars the gap is 20, and 20 divided by 80 equals 0.25, a 25% decrease.



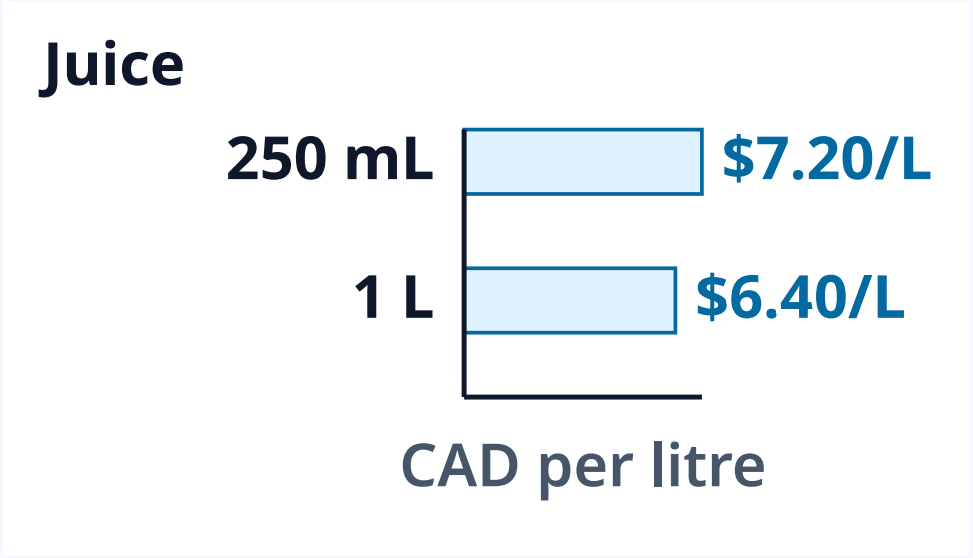
Three cans or five cans

- A sales promotion can sell three cans for 4.50 dollars or five cans for 7.00 dollars.
- Three cans at 4.50 dollars cost 1.50 dollars each, because 4.50 divided by 3 equals 1.50.
- Five cans at 7.00 dollars cost 1.40 dollars each, because 7.00 divided by 5 equals 1.40.



Small-print units and a juice bottle

- Deli meat at 2 dollars per 100 grams is 2 divided by 0.1, which equals 20 dollars per kilogram.
- That matches a tag that already reads 20 dollars per kilogram.
- Juice at 250 millilitres is 0.25 litres.
- Dividing 1.80 by 0.25 equals 7.20 dollars per litre.
- A 1 litre bottle at 6.40 dollars costs 6.40 dollars per litre.
- A brush that moves from 48 dollars to 36 dollars falls by 12, and 12 divided by 48 equals a 25% decrease.



Shelf price check

- Find the unit price of each rice bag and name the bag with the lower unit price.
- State the percent change from 40 dollars to 50 dollars and say whether it is an increase.



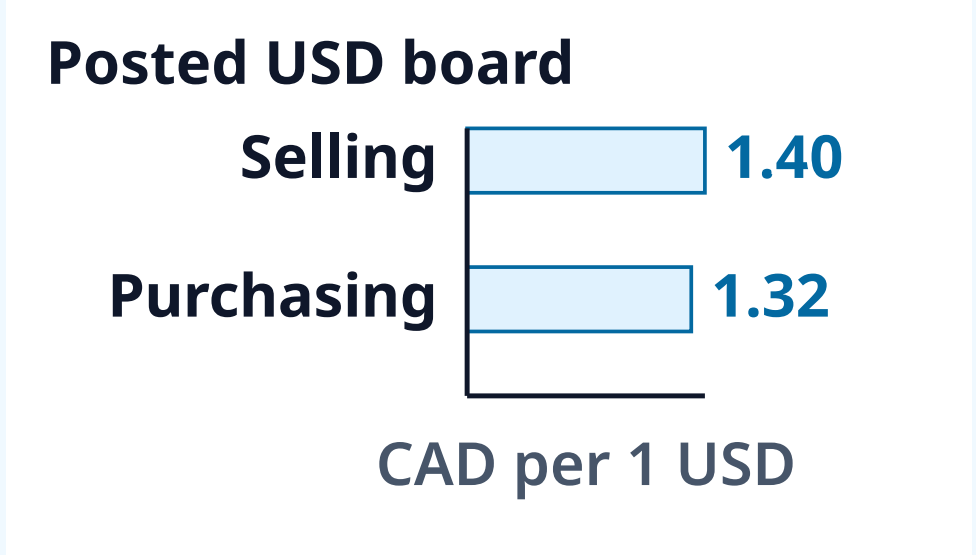
Mathematics 10-3

Unit N: Money and income

Currency exchange at the posted rate

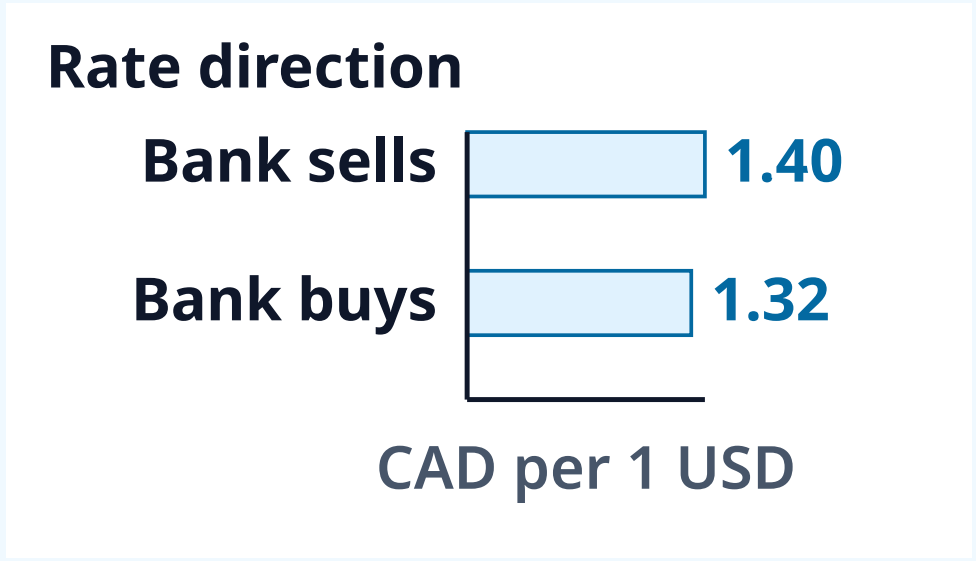
A posted board before a trip

- A teller trainee reads a posted sample board: the bank sells 1 USD for 1.40 CAD and buys 1 USD for 1.32 CAD.
- A customer who wants 50 US dollars pays at the selling rate.
- Selling those dollars back uses the purchasing rate.



Buying and selling rates

- The selling rate is the rate the institution uses when it sells foreign currency to the customer.
- The purchasing rate is the rate the institution uses when it buys foreign currency from the customer.
- Currency exchange multiplies the foreign amount by the rate for that direction, and a rate table shows that multiplication in two columns.



Key terms

Selling rate

The rate the institution uses when it sells foreign currency to the customer. The customer pays this many Canadian dollars per one foreign unit. (CAD per 1 foreign unit)

Purchasing rate

The rate the institution uses when it buys foreign currency from the customer. The customer receives this many Canadian dollars per one foreign unit. (CAD per 1 foreign unit)

Currency exchange

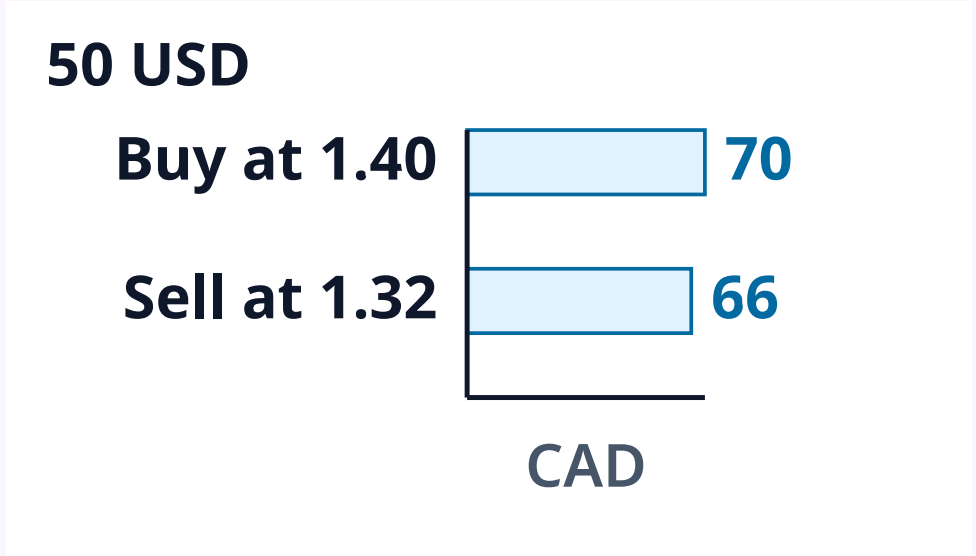
A conversion between Canadian dollars and a foreign currency by multiplying by the rate for that direction.

Rate table

A two-column chart that lists a foreign amount beside the Canadian dollars for that amount at one stated rate.

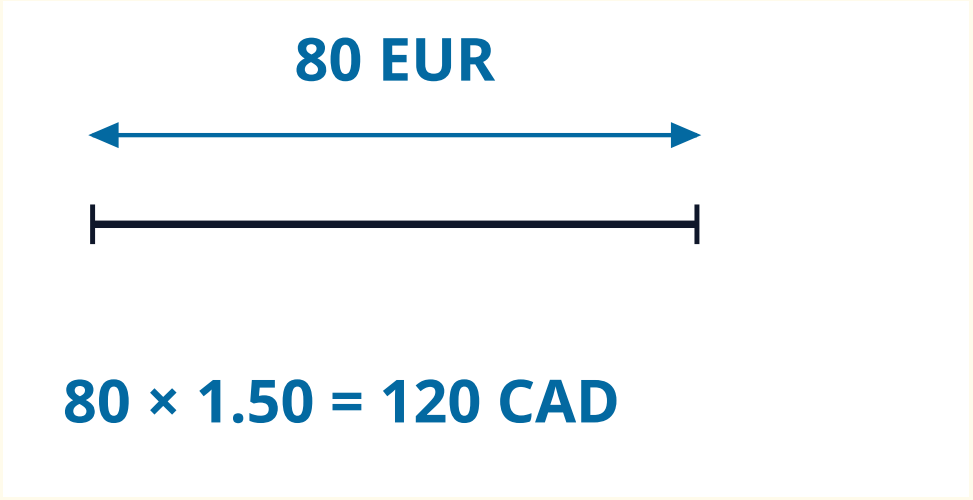
Fifty US dollars out and back

- Buying 50 USD at the selling rate 1.40 costs 50 times 1.40, which equals 70 CAD.
- Selling 50 USD back at the purchasing rate 1.32 yields 50 times 1.32, which equals 66 CAD.
- The gap of 70 minus 66, which equals 4 CAD, is the institution’s margin on that round trip.



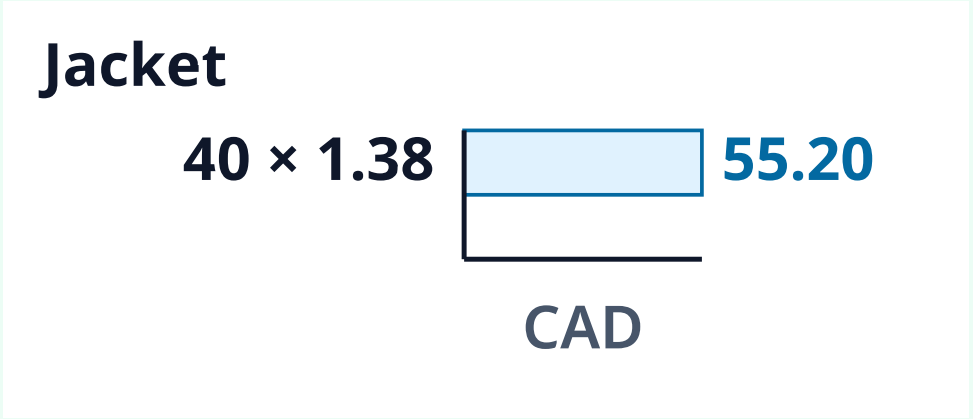
Eighty euros at a selling rate of 1.50

- A selling rate of 1.50 CAD per EUR means each euro costs 1.50 Canadian dollars.
- Eighty euros cost 80 times 1.50.
- Multiplying 80 by 1 equals 80, multiplying 80 by 0.50 equals 40, and 80 plus 40 equals 120 CAD.



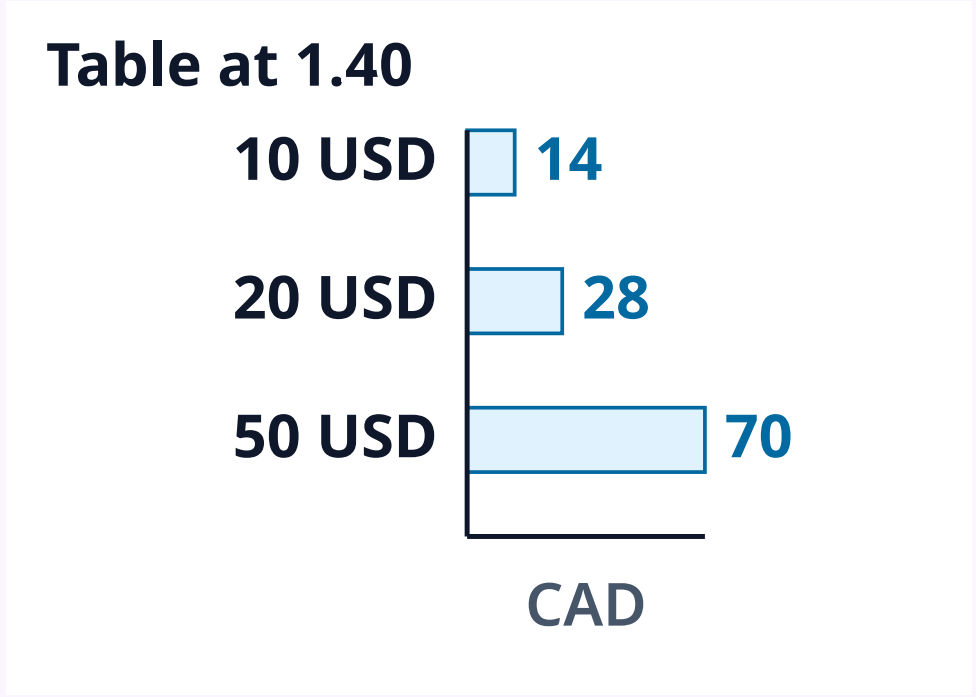
A jacket priced in US dollars

- A jacket costs 40 USD and the selling rate for this question is 1.38 CAD per USD.
- Forty times 1.38 equals 55.20, so the jacket costs 55.20 CAD.
- A purchasing rate of 1.30 would price the jacket as if the customer were selling USD to the bank.



Estimating with a rate table

- While abroad, round the selling rate and multiply before you pay, so a 1.40 rate stays 1.4 times the foreign price in CAD.
- Ten times 1.40 equals 14, twenty times 1.40 equals 28, and fifty times 1.40 equals 70.
- Those rows price 10 USD at 14 CAD, 20 USD at 28 CAD, and 50 USD at 70 CAD.
- Rounding 1.38 to 1.40 gives 40 times 1.40.
- Forty times 1 equals 40 and 40 times 0.40 equals 16, so the estimate is 56 CAD.



Board check at the counter

- Find the cost of 50 USD at the selling rate 1.40 and the receipt from selling 50 USD at the purchasing rate 1.32.
- Find the cost of 80 EUR at a selling rate of 1.50.



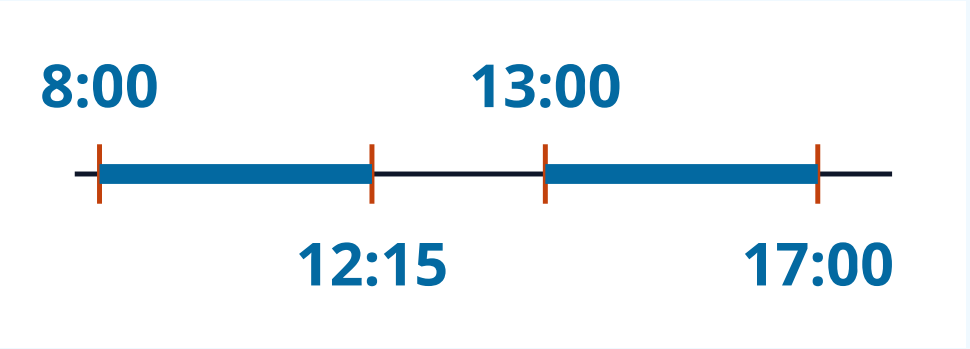
Mathematics 10-3

Unit N: Money and income

Methods of earning income

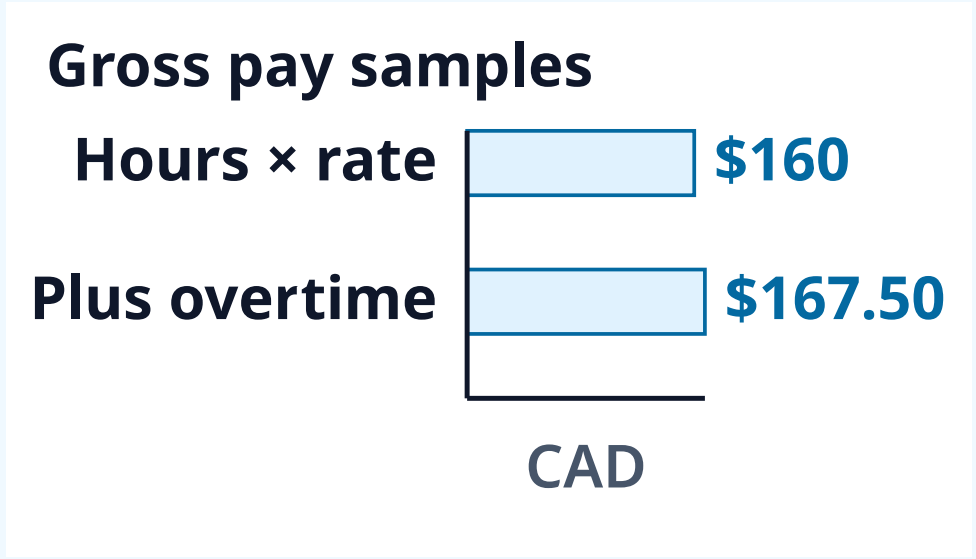
A time clock and a tip jar

- A welding apprentice clocks 8:00 to 12:15 and 13:00 to 17:00, and a server on another shift counts a wage plus tips.
- The earning method decides which numbers enter the pay: hours, sales, finished pieces, or one contract price.



Methods of earning income

- Methods of earning include hourly wage, wage plus tips, salary, commission, contract, piecework, bonus, and shift premium.
- A welder is paid an hourly wage, and a server is paid a wage plus tips.
- A salesperson earns commission, and office staff earn a salary.
- Roofing pays by contract, sewing pays by piecework, and a night shift adds a premium.
- Straight-time gross pay uses hours times the hourly rate.
- Overtime pay uses time and a half or double time on the extra hours.



Key terms

Hourly wage

Pay that counts each hour worked, including overtime hours at the overtime factor the job states.

Overtime

Hours beyond the stated threshold, paid at time and a half or at double time.

Commission

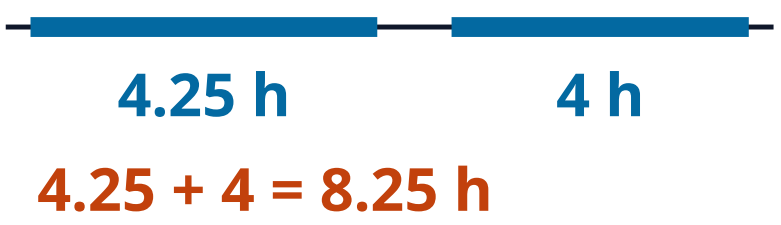
Pay that is a percent of sales, either as a single rate or as a base plus that percent.

Gross pay

The pay counted from the earning method before the deductions on a later stub. (hours × rate for straight time)

A time card in decimal hours

- From 8:00 to 12:15 is 4 hours 15 minutes, and 15 divided by 60 equals 0.25, so that block is 4.25 hours.
- From 13:00 to 17:00 is 4 hours, and 4.25 plus 4 equals 8.25 hours on the card.
- Fifteen minutes is 0.25 hour because 60 minutes equal 1 hour, while 0.15 hour is 9 minutes.



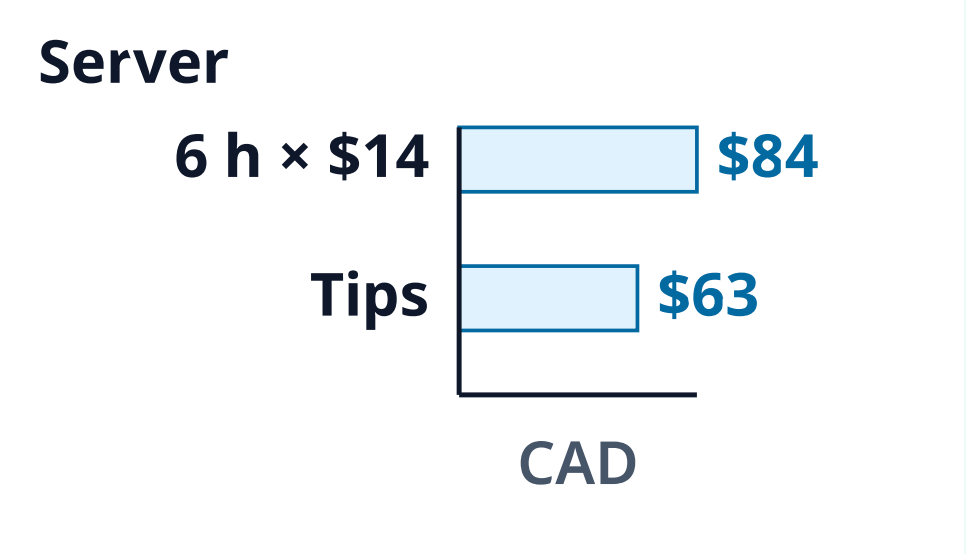
Time and a half after eight hours

- The rate is 20 dollars per hour, with overtime after 8 hours at time and a half.
- Regular pay is 8 times 20, which equals 160.
- Overtime pay starts with 0.25 times 20, which equals 5.
- Then 5 times 1.5 equals 7.50.
- Add 160 and 7.50 to get a gross pay of 167.50.



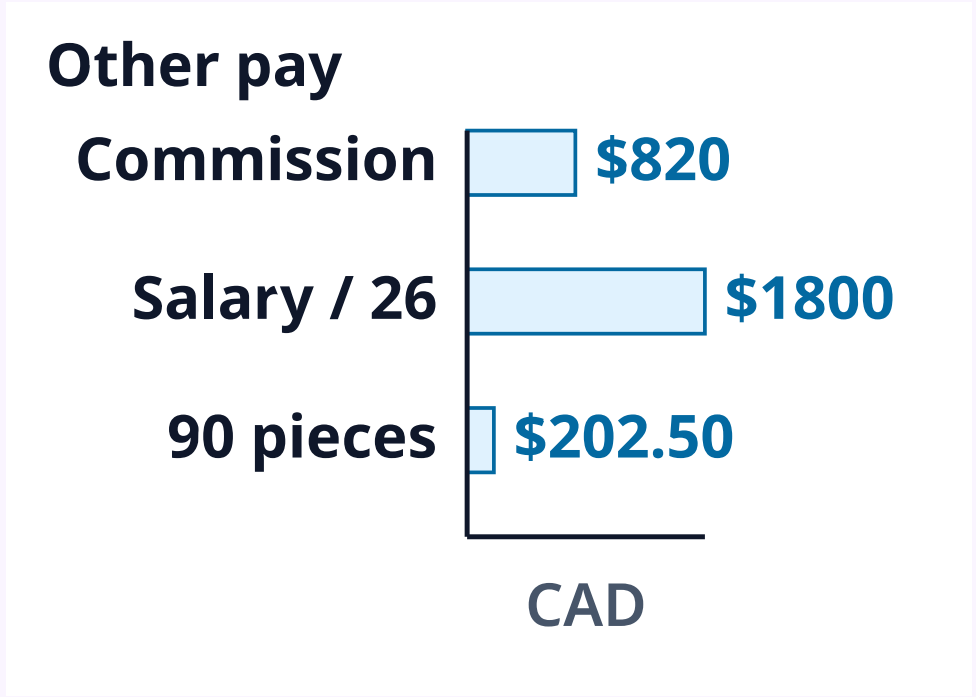
A server’s wage plus tips

- A server works 6 hours at 14 dollars and receives 63 dollars in tips.
- The wage is 6 times 14, which equals 84 dollars.
- Gross pay is 84 plus 63 in tips, which equals 147 dollars.



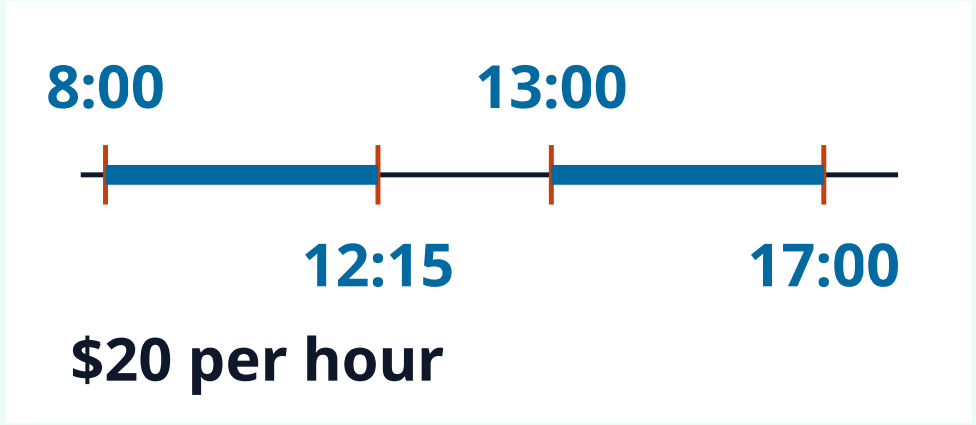
Salary and commission

- Double time for 5 hours at 18 dollars starts with 5 times 18, which equals 90.
- Then 90 times 2 equals 180 dollars of double-time pay.
- Four percent of 8000 dollars is 0.04 times 8000, which equals 320.
- A base of 500 dollars plus 320 equals 820.
- Six percent of 3500 dollars is 0.06 times 3500, which equals 210.
- Dividing 46800 dollars by 26 gives 1800, dollars because 26 times 1800 equals 46800.
- Ninety pieces at 2.25 dollars start with 90 times 2, which equals 180, and 90 times 0.25 equals 22.50.
- Adding 180 and 22.50 equals 202.50 dollars.
- A painting contract of 1800 dollars is the gross pay for that job.



Card and formula check

- Write the welder card as a decimal number of hours.
- Find the gross pay for 8.25 hours at 20 dollars with over-time after 8 hours at time and a half.



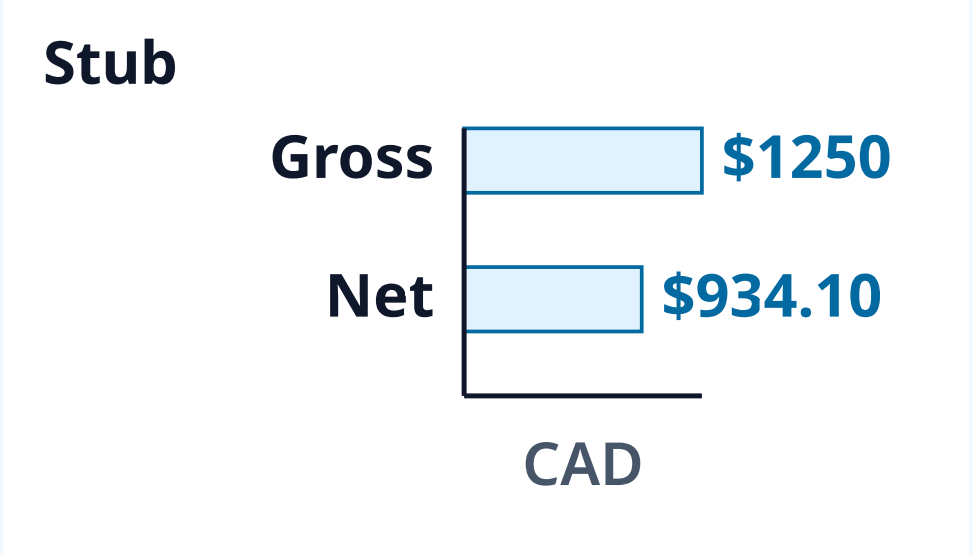
Mathematics 10-3

Unit N: Money and income

Gross pay and net pay

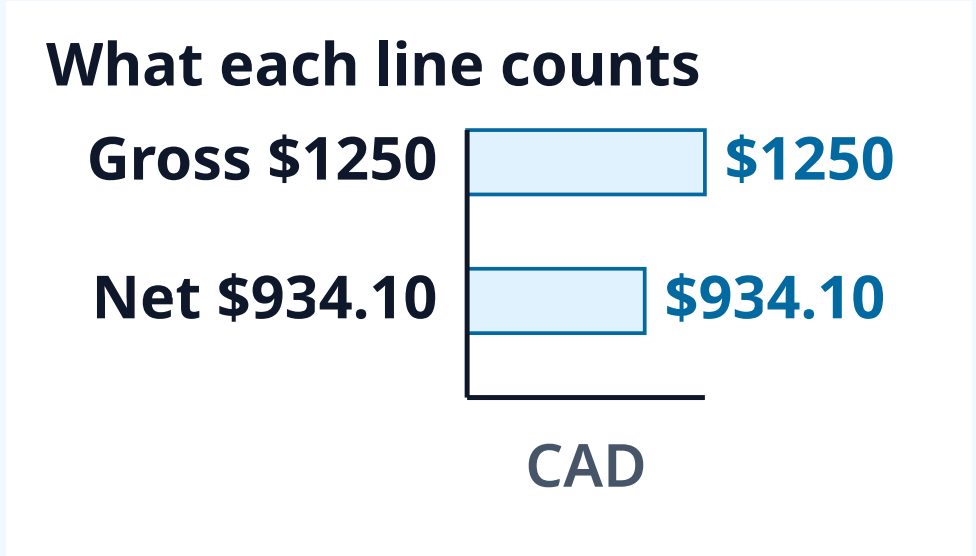
The deposit is the last line

- A co-op student opens a pay stub whose top line is gross pay of 1250 dollars.
- The deposit matches the last line, after the stub's given deductions have been subtracted one at a time.



What gross pay and net pay count

- Gross pay counts the pay before the listed deductions.
- Net pay counts the pay after those listed deductions have been subtracted.
- The CPP, EI, income tax, and health plan figures in this lesson are the amounts the question gives on the stub.



Key terms

Gross pay

The pay counted before the listed deductions are subtracted.

Net pay

The pay counted after the listed deductions have been subtracted.

Deduction

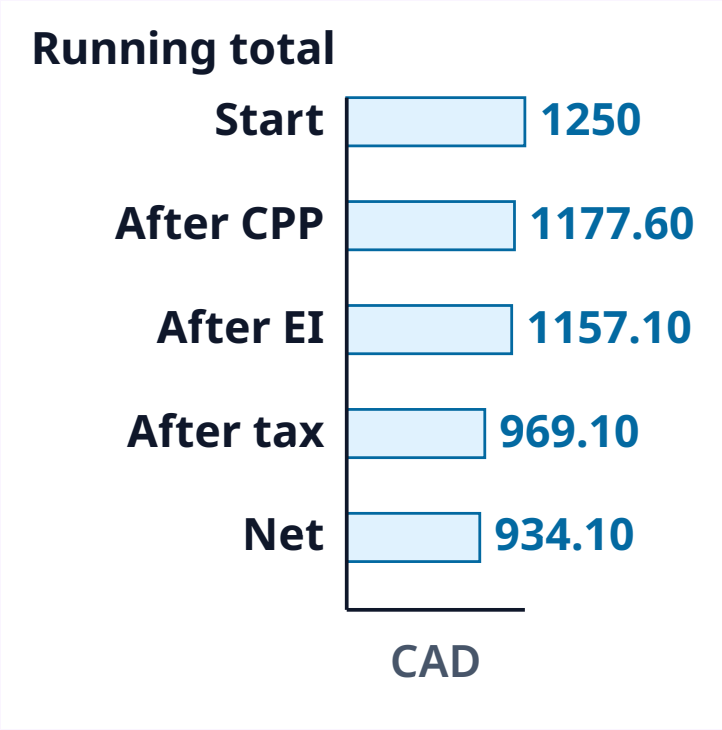
An amount the question lists to subtract from gross pay, such as the given CPP, EI, income tax, or health plan figure.

Pay stub

The slip that lists gross pay, each given deduction, and the net pay that remains.

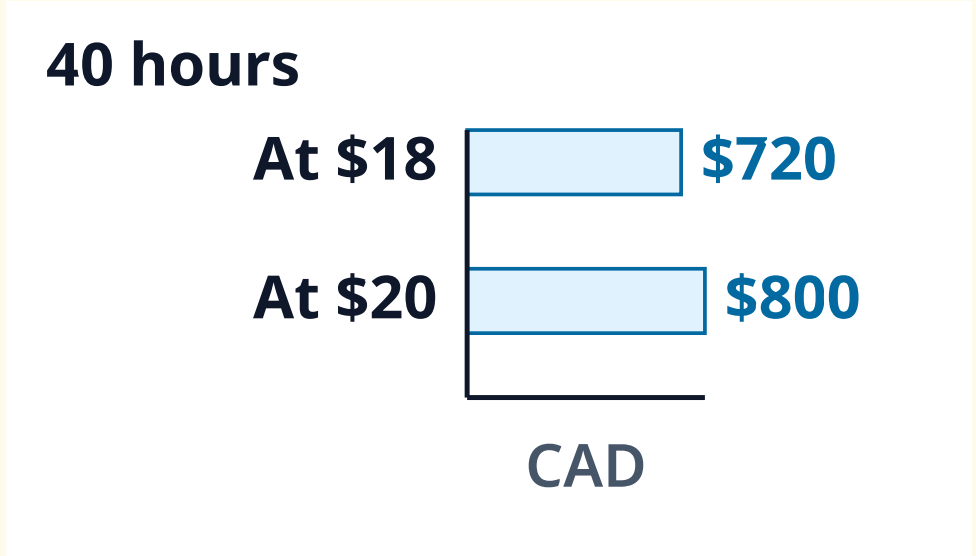
A running total down the stub

- Start at gross pay 1250 dollars and subtract CPP 72.40 dollars.
- That 1250 minus 72.40 equals 1177.60.
- Subtract EI 20.50 dollars.
- That 1177.60 minus 20.50 equals 1157.10.
- Subtract income tax 188 dollars.
- That 1157.10 minus 188 equals 969.10.
- Subtract the health plan 35 dollars.
- That 969.10 minus 35 equals 934.10.
- Net pay on this stub is 934.10 dollars.



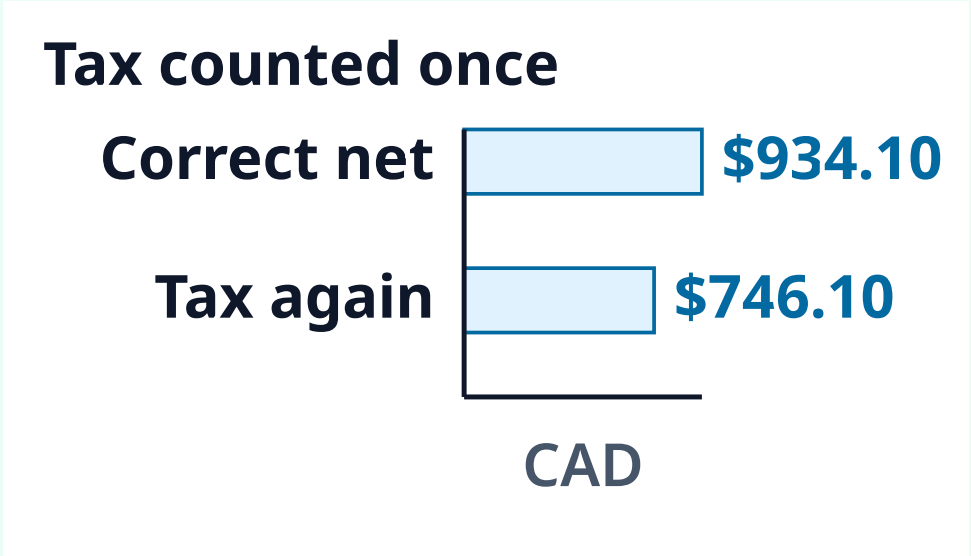
A what-if on the hourly rate

- A simple table compares 40 hours at 18 dollars with 40 hours at 20 dollars.
- Multiplying 40 by 18 equals 720, and multiplying 40 by 20 equals 800.
- The gross rises by 800 minus 720, which equals 80, before any deduction change.



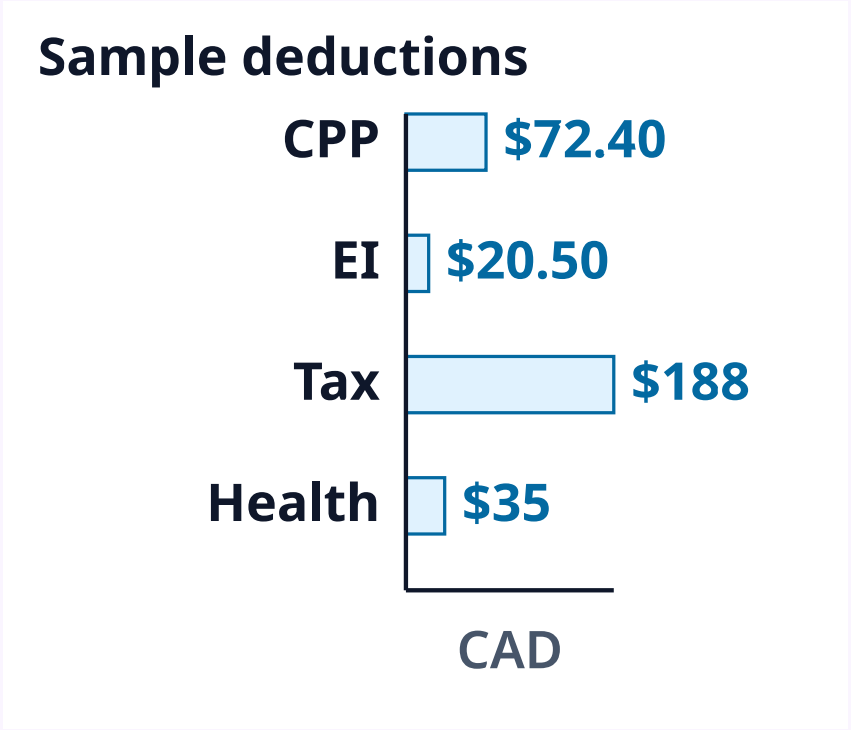
Income tax subtracted a second time

- The corrected net pay is 934.10 after the 188 dollars tax has already been subtracted once.
- Subtracting that 188 dollars again produces 934.10 minus 188.
- That second subtraction double-counts the tax, and the corrected net stays 934.10.



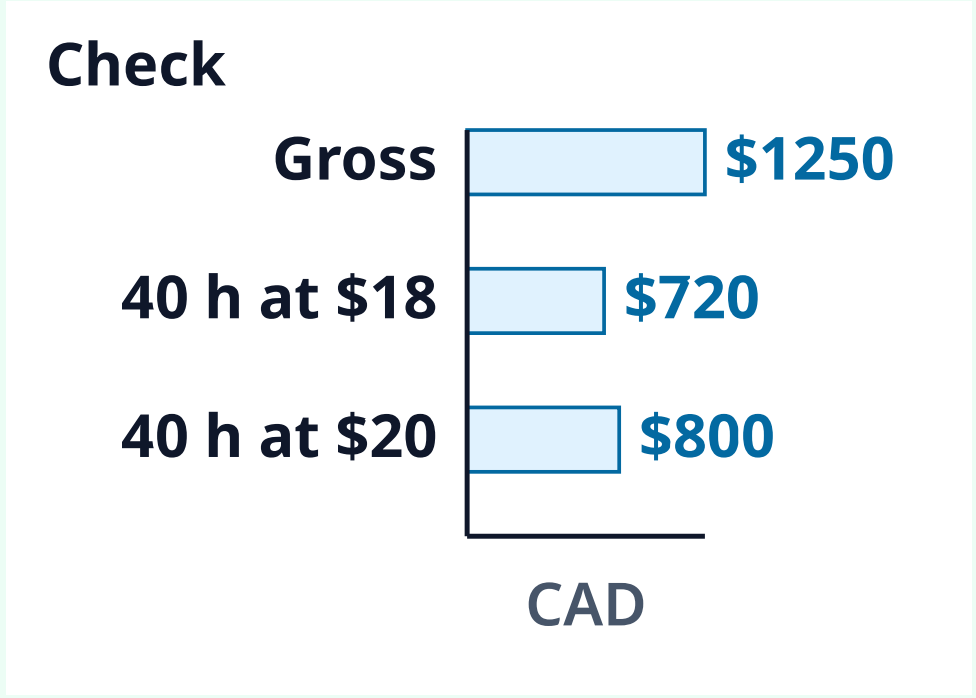
Other lines a stub can list

- Uniforms, union dues, charitable donations, and a payroll tax can appear as further given deductions.
- Each of those lines is another subtraction when the question lists it.
- Gross pay still counts the pay before the lines, and net pay still counts what remains after the lines the question includes.



Stub check

- Find the net pay for gross 1250 dollars with the four given deductions 72.40 dollars, 20.50 dollars, 188, dollars and 35 dollars.
- State the gross-pay rise from 40 hours at 18 dollars to 40 hours at 20 dollars.



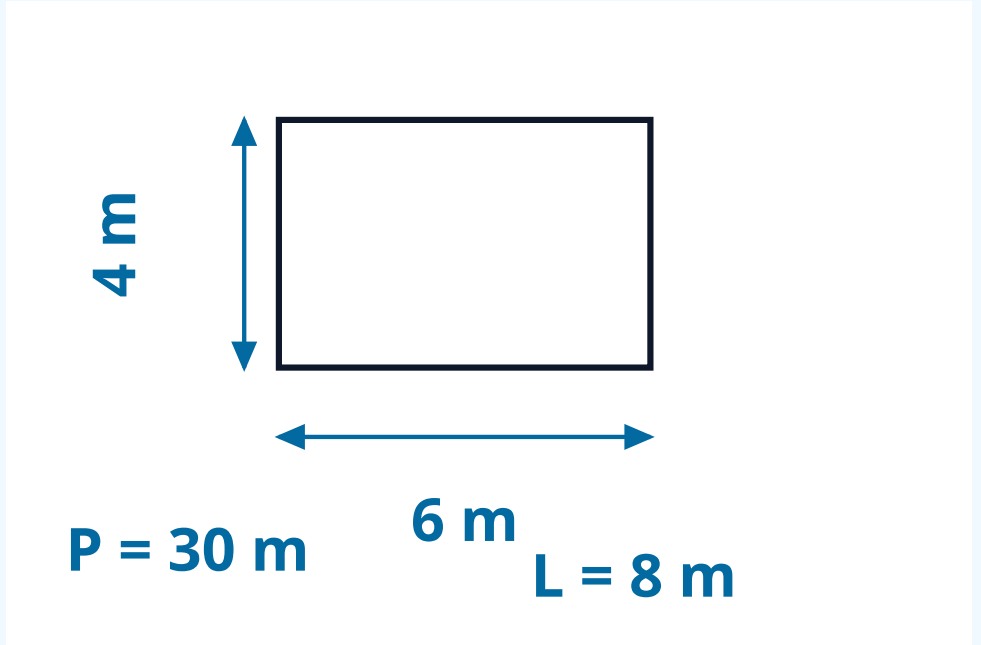
Mathematics 10-3

Unit A: Algebra and working with formulas

Workplace formulas

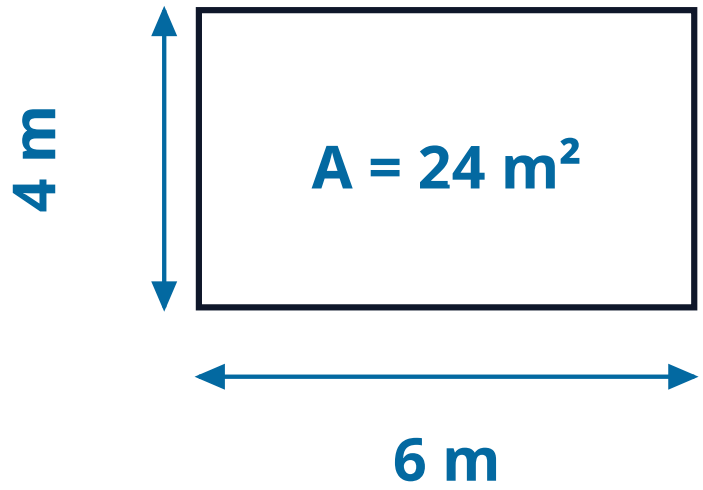
Coating area and rail length

- A painter has a panel 6 metres by 4 metres and wants the area before ordering coating.
- A fence crew has a perimeter of 30 metres and a length of 8 metres, and the width is the measure still wanted.



Substitution and isolating a variable

- Substitution replaces each letter in a formula with a known measure and then follows the operations.
- On a panel, A means area, l means length, and w means width.
- A = l times w = 6 times 4 = 24 square metres.
- A rearrangement isolates the letter a worker needs.
- W = P/2 - L is the rearrangement of P = 2(L + W).
- Equivalent forms name one relationship.
- A = l w and w = A/l agree because 24 divided by 6 equals 4 and 6 times 4 equals 24.
- The radius is half a diameter.



Key terms

formula

A formula is a rule that links workplace measures through letters and operations. (A = l w)

substitution

Substitution is the replacement of each letter by a known measure, followed by the operations written in the formula. (A = 6 × 4)

rearrangement

A rearrangement is a form of a formula in which the letter a worker needs stands alone. (W = P/2 - L)

equivalent forms

Equivalent forms are two writings of one relationship, confirmed when the same measures satisfy both writings. (A = l w and w = A/l)

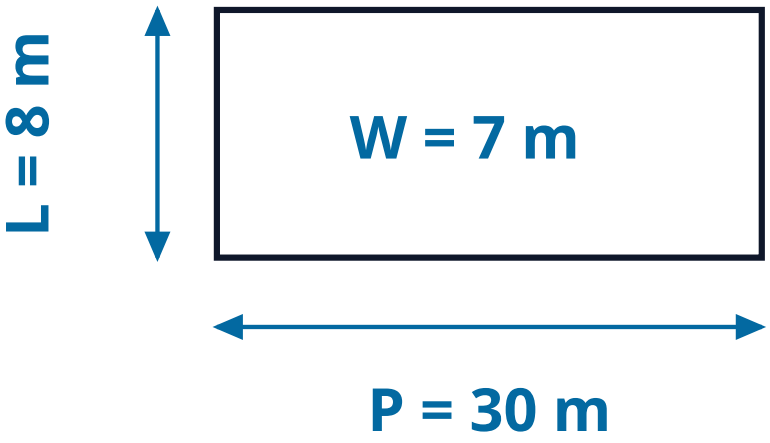
Key terms

radius

The radius is half a diameter, and it is the length taken by the letter r in a circle area formula. (r = d/2)

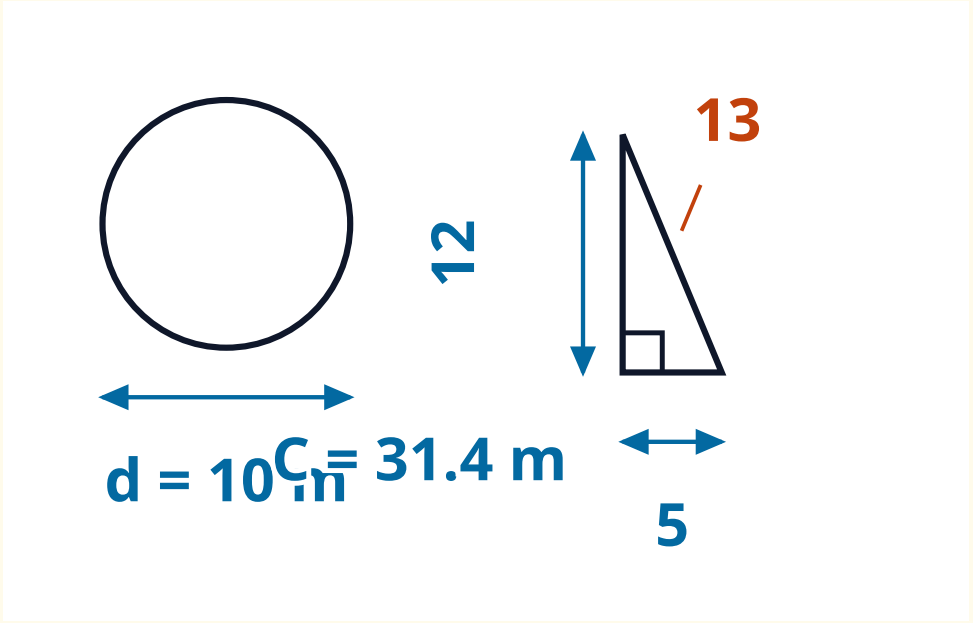
Area and width from formulas

- Direct substitution in A = l times w uses a length of 6 metres and a width of 4 metres.
- Six times 4 equals an area of 24 square metres.
- The equivalent form w = A/l returns 24 divided by 6 equals 4, and substituting those edges back gives 6 times 4 equals 24.
- With P = 30 metres and L = 8 metres, P/2 equals 15.
- W = 15 minus 8 = 7 metres, from the form W = P/2 - L.



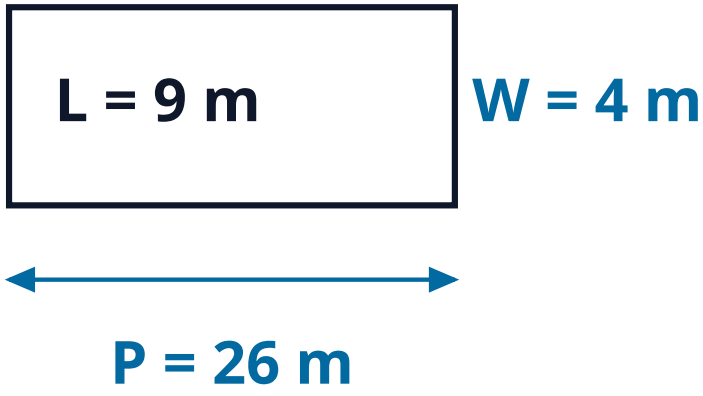
Formulas in trades

- The rim formula is $C = \pi d$, with $C = 31.4$ metres and $\pi = 3.14$.
- Divide 31.4 by 3.14.
- Because 3.14 times 10 equals 31.4, the diameter is 10 metres.
- The brace has hypotenuse 13 metres and one leg of 5 metres.
- Square 13 to get 169, and square 5 to get 25.
- Subtract 169 minus 25 to get 144.
- The square root of 144 is 12, so the other leg is 12 metres.
- The pay envelope is 720 dollars for 36 hours.
- Divide 720 by 36 to get 20 dollars per hour.



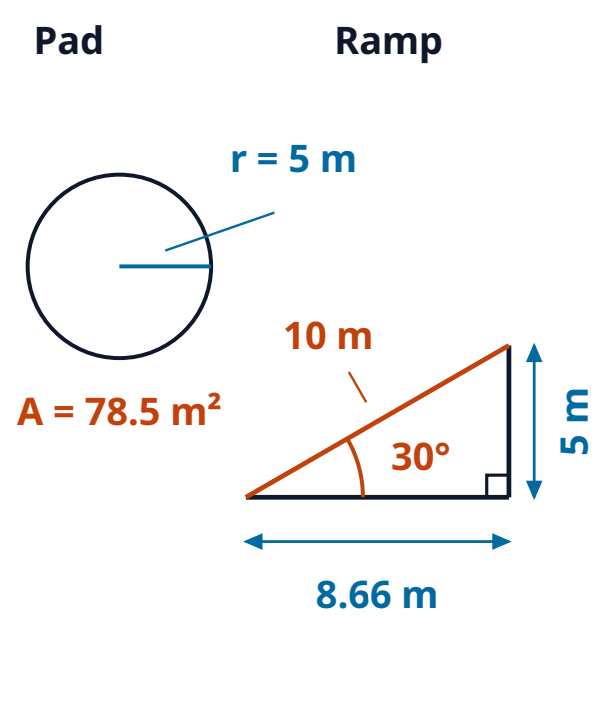
A fence question

- The comparison fence has a perimeter budget of 26 metres and one side already set at 9 metres.
- Half of 26 is 13, and 13 minus 9 is 4, so the adjacent side is 4 metres from W = P/2 - L.
- Your own question needs a new perimeter, a new known side, and the same two operations.



Formula errors in trades

- On a ramp, opposite = hypotenuse × sin(angle).
- A ramp of 10 metres at 30 degrees has sin 30 degrees = 0.5, so the opposite side is 10 × 0.5 = 5 metres, and the base is 8.66 metres.
- A painter uses area, and a payroll clerk uses rate = pay divided by hours.
- A framer rearranges the Pythagorean formula, and a machinist uses circumference to recover a diameter.
- A common mistake is to put the 10 m diameter in place of r. That gives 3.14 × 100 = 314, four times too big.
- The radius is 5 metres, and the area is 3.14 × 25 = 78.5 square metres.



Yard width and pad area

- Find the width of a storage yard with a perimeter of 30 metres and a length of 8 metres.
- Correct the area of a circular pad with a diameter of 10 metres when the ticket squared that diameter in $A = \pi r^2$.

